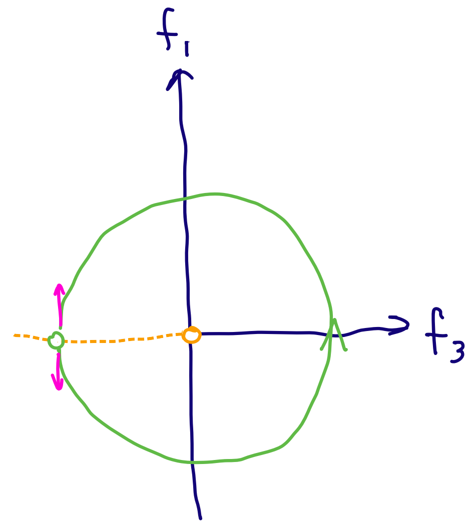
$$H_{herm} = \begin{bmatrix} f_3 & f_1 \\ f_1 & -f_3 \end{bmatrix}$$

$$\det\left(H_{herm} - I\omega\right) = \det\begin{bmatrix} f_3-\omega & f_1 \\ f_1 & -f_3-\omega \end{bmatrix}$$

$$= (f_3-\omega)(-f_3-\omega) - f_1^2$$

$$= \omega^2 - (f_3^2 + f_1^2)$$



$$\Rightarrow \quad \omega_\pm = \pm\sqrt{f_3^2 + f_1^2}$$

$$\begin{bmatrix} f_3-\omega_+ & f_1 \\ f_1 & -f_3-\omega_+ \end{bmatrix} v_+ = 0$$

$$\begin{bmatrix} f_3-\sqrt{} & f_1 \\ f_1 & -f_3-\sqrt{} \end{bmatrix} \xrightarrow{\cancel{f_1=0,\ f_3\leq 0}}$$

$$\begin{bmatrix} (f_3-\sqrt{})(-f_3-\sqrt{}) & f_1(-f_3-\sqrt{}) \\ f_1 & -f_3-\sqrt{} \end{bmatrix} \longrightarrow \begin{bmatrix} f_1^2 & -f_1f_3-f_1\sqrt{} \\ f_1 & -f_3-\sqrt{} \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 0 & 0 \\ f_1 & -f_3-\sqrt{} \end{bmatrix}$$

$$\Rightarrow \quad V_+ = \begin{bmatrix} f_3 + \sqrt{f_3^2 + f_1^2} \\ \\ f_1 \end{bmatrix} \underset{\substack{\left\{\begin{array}{l} f_3 = \cos\theta \\ f_2 = \sin\theta \end{array}\right. \\ -\pi < \theta < \pi}}{=} \begin{bmatrix} \cos\theta + 1 \\ \\ \sin\theta \end{bmatrix}$$

Observe that when $\theta \to (-\pi)+$, we have $\cos\theta + 1 \to 0+$ and $\sin\theta \to 0-$
whereas when $\theta \to \pi-$, we have $\cos\theta + 1 \to 0+$ and $\sin\theta \to 0+$.

$$\lim_{\theta \to (-\pi)+} \frac{V_+}{|V_+|} = \lim_{\theta \to (-\pi)+} \begin{bmatrix} \dfrac{\cos\theta + 1}{-\sin\theta} \\ \\ -1 \end{bmatrix} \Big/ |\ \ |$$

As $\theta \to (-\pi)+$,

$$\frac{\cos\theta + 1}{+\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \longrightarrow 0$$

$$\left.\begin{array}{l} \Rightarrow \displaystyle\lim_{\theta \to (-\pi)+} \frac{V_+}{|V_+|} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\ \\ \displaystyle\lim_{\theta \to \pi-} \frac{V_+}{|V_+|} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{array}\right\} \text{Half Möbius band!}$$