Robust Model Reconstruction Based on the Topological Understanding of Point Clouds Using Persistent Homology

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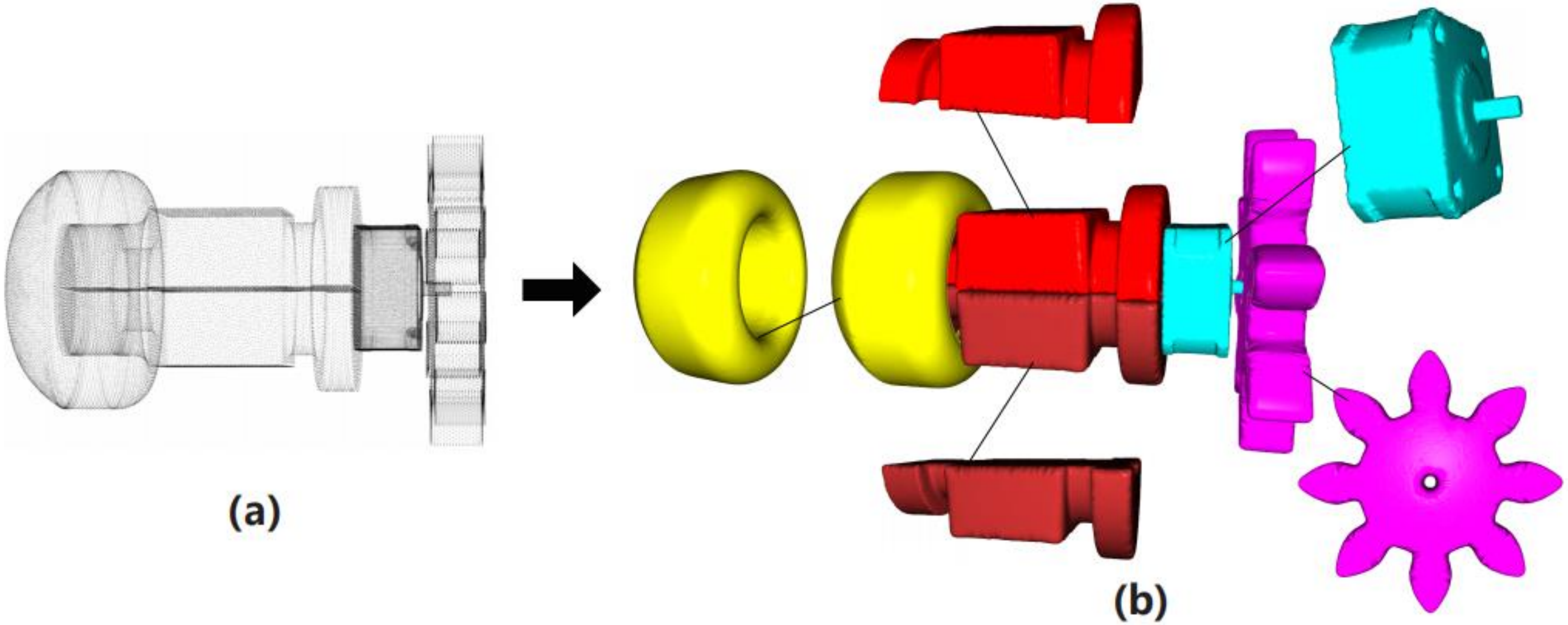
Robust Model Reconstruction

➢ Surface reconstruction: reconstruct a surface from a sampling point cloud.

➢ Traditional reconstruction methods usually focus on single surface reconstruction. But if the given point cloud represents a complex model composed of multiple components, it can be challenging to reconstruct the whole model directly.

➢ Our method can reconstructe a complex model composed of multiple components from point cloud directly. These components are multiple closed surfaces with shared regions.



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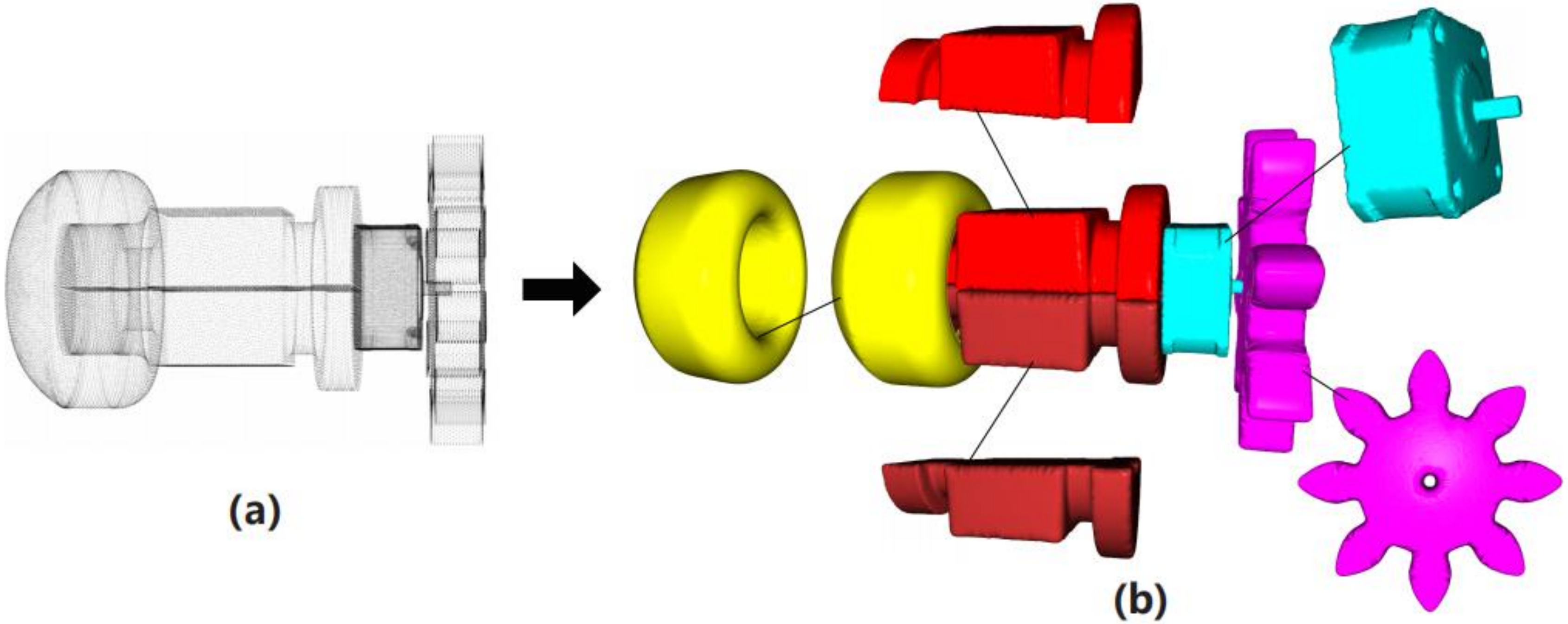
Robust Model Reconstruction

- In other words, we develop a novel method for automatically reconstructing closed surfaces with sharing regions from unorganized point clouds.
- By integrating topological analysis with geometric processing techniques, we address the challenges of reconstructing multi-component models from point clouds with noise.



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Topological Understanding by 2-PD

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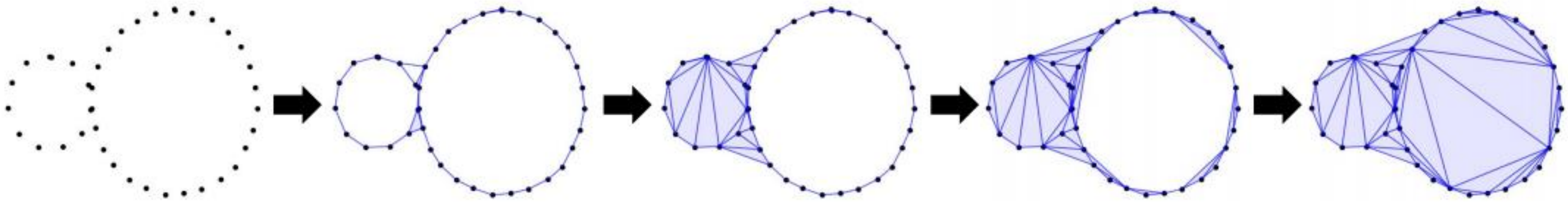
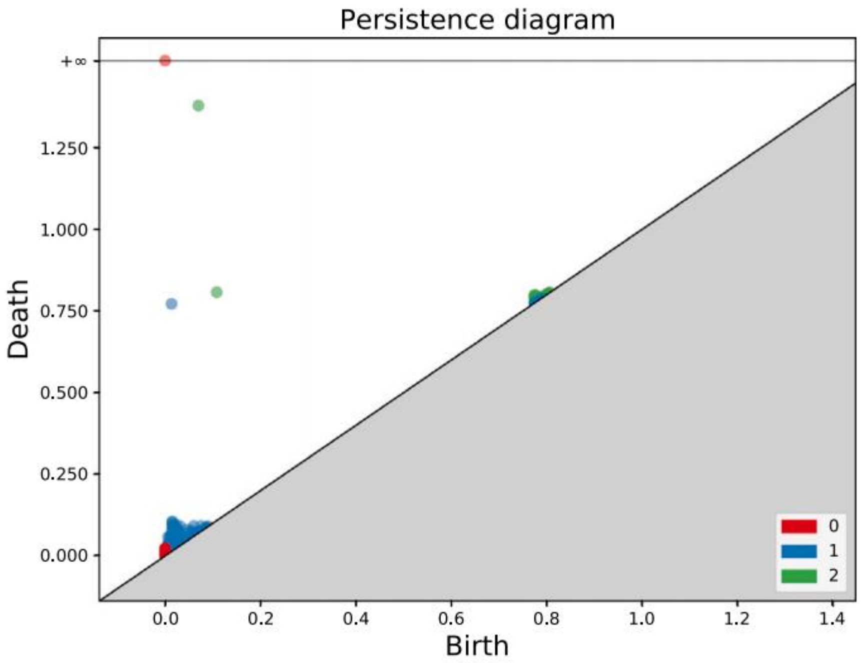
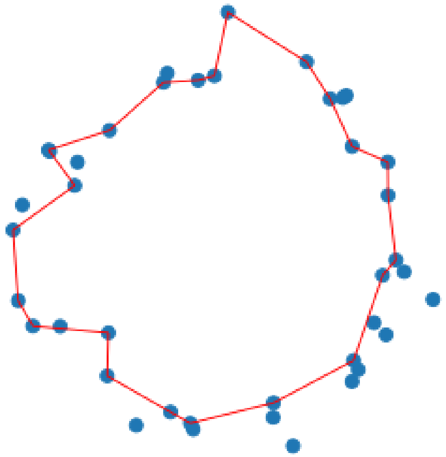
- Topological Understanding is provided by **persistent homology**, a powerful tool for analyzing the topology of point cloud. It is computed by constructing a sequence of simplicial complex from the point cloud.
- **Alpha complex** is a certain type of simplicial complex. The **alpha filtration** is a sequence of alpha complex (a certain type of simplicial complex), it can be constructed by setting an increasing parameter with adding simplices in the simplicial complex.
- We define a simplex as **positive** if its addition creates a cycle, thereby giving birth to a new homology class. Otherwise, we define it as **negative**.
- We compute the alpha filtration of the given point cloud, and get topological understanding from persistent homology (2-PD).



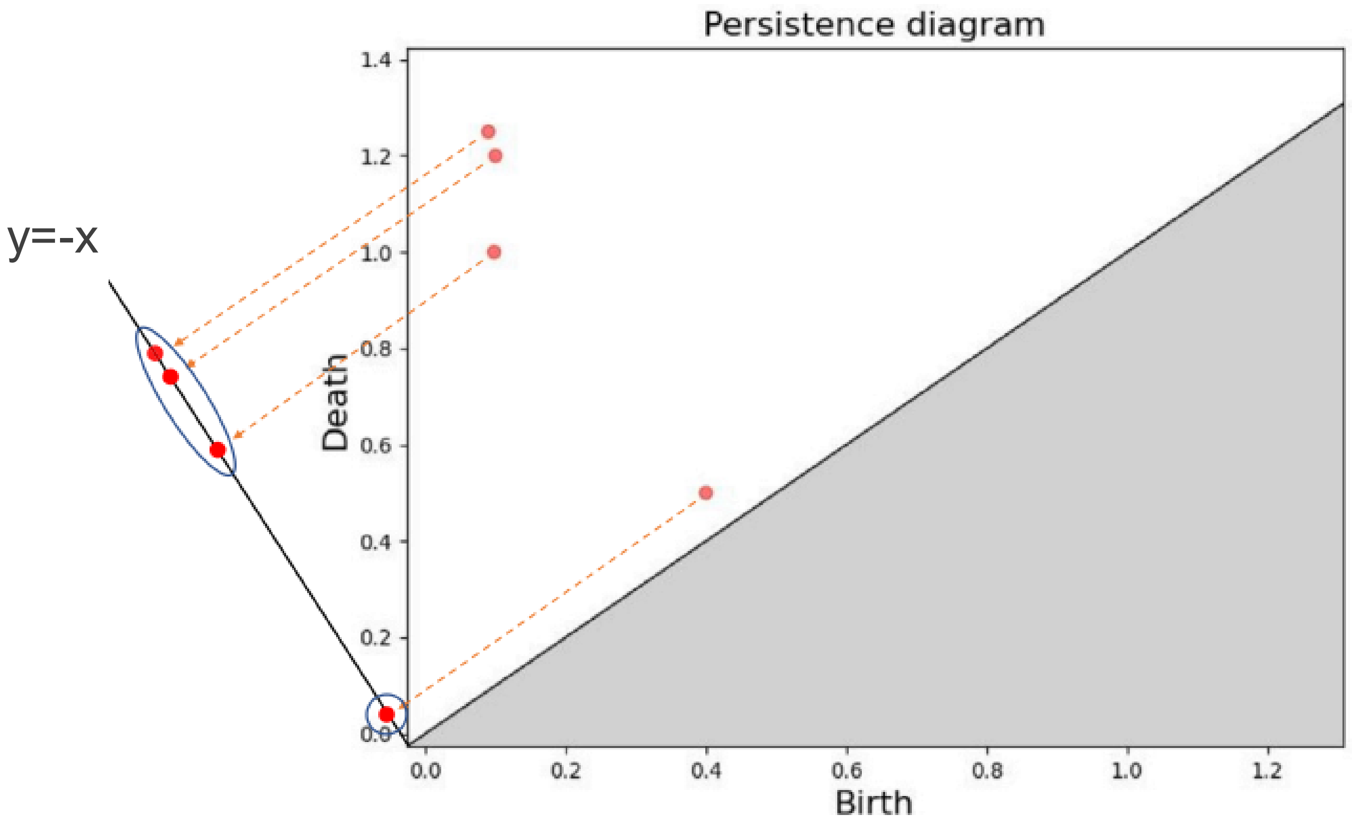
- **Persistence Diagram (PD)** is a visualization tool of persistent homology. Each point in a PD records a topological feature in the point cloud. The x- and y-coordinates are the birth and death time (b_i, d_i) of these features respectively.
- The persistence pf a point in PD is $d_i - b_i$, the lifespan of the topological feature.
- Usually, the points with great persistence are regarded as **significant**, corresponding to significant topological features in the point cloud. And the geometric objects of these features are called **representative cycles** of points in PD.





Persistence Diagram

- By clustering the points in 2-PD and find significant points, we get the number of components in the model.
- Then we can extract the representative 2-cycles of significant points in 2-PD.

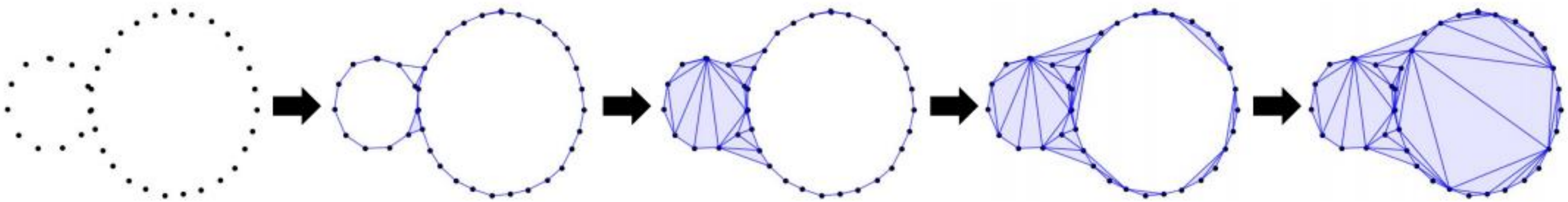


Extract Representative 2-cycles

- After getting significant points in 2-PD, we can extract the **volume-optimal 2-cycles** (consist of 2-simplices) of these significant points. These cycles are represented as closed triangular surfaces.
- If the bounadry of a 3-chain is exactly the volume-optimal 2-cycle, we call the 3-chain a **persistent volumn**.
- This problem can be regarded as solving an optimization problem, where b_i and d_i are the birth time and death time of the 2-d holes, and σ are the 3-simplices we considered.

$$\text{minimize } \|z\|_1 \text{ subject to}$$
$$z = \sigma_{d_i} + \sum_{\sigma_k \in \mathcal{F}_3,\, \alpha_k \in \mathbb{Z}_2} \alpha_k \sigma_k,$$
$$\tau^*(\partial z) = 0, \quad \forall \tau \in \mathcal{F}_2$$
$$\sigma_{b_i}^*(\partial z) \neq 0.$$

$$\mathcal{F}_q = \{\sigma : q\text{-simplex}, \sigma \in \mathrm{Alp}(P, d_i) - \mathrm{Alp}(P, b_i)\}$$



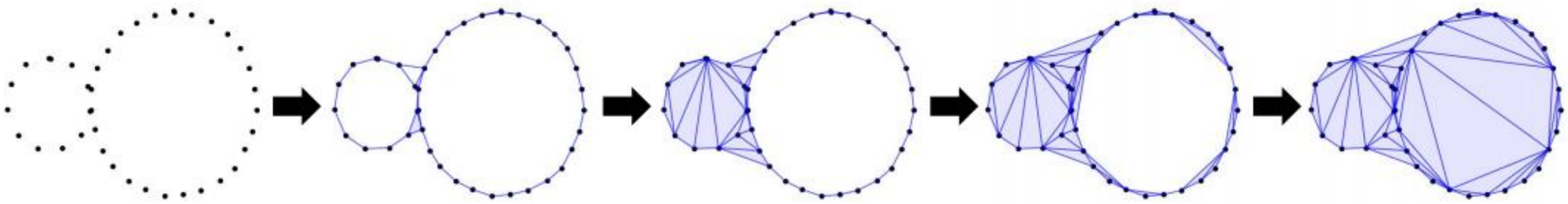
Extract Representative 2-cycles

- Suppose z is the persistent volumn we want (a 3-chain), this optimization problem has three constraints:

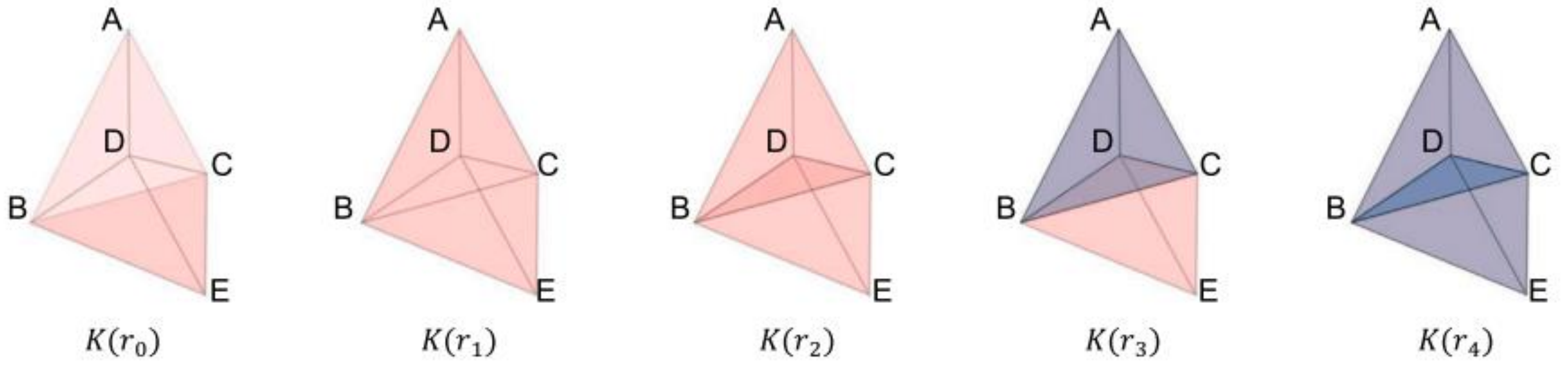
1. z must include the negative 3-simplex that kills the representative 2-cycle, and other 3-simplices can only be added during b_i and d_i ;
2. The boundary of z does not include the 2-simplices added during b_i and d_i ;
3. The boundary of z must contain the positive 2-simplex that give birth to the 2-cycle.

$$\text{minimize } \|z\|_1 \text{ subject to}$$
$$z = \sigma_{d_i} + \sum_{\sigma_k \in \mathcal{F}_3,\, \alpha_k \in \mathbb{Z}_2} \alpha_k \sigma_k,$$
$$\tau^*(\partial z) = 0, \quad \forall \tau \in \mathcal{F}_2$$
$$\sigma^*_{b_i}(\partial z) \neq 0.$$

$$\mathcal{F}_q = \{\sigma : q\text{-simplex}, \sigma \in \mathrm{Alp}(P, d_i) - \mathrm{Alp}(P, b_i)\}$$



Example



$$\text{minimize } \|z\|_1 \text{ subject to}$$
$$z = \sigma_{d_i} + \sum_{\sigma_k \in \mathcal{F}_3, \alpha_k \in \mathbb{Z}_2} \alpha_k \sigma_k,$$
$$\tau^*(\partial z) = 0, \quad \forall \tau \in \mathcal{F}_2$$
$$\sigma^*_{b_i}(\partial z) \neq 0.$$

- In this filtration, the simplicial complex $K(r_0)$ contains five 2- simplices: [ABD], [ACD], [BCE], [BDE], [CDE].
- In $K(r_1)$, the 2-simplex [ABC] is added, generating a 2-cycle:

  [ABC] + [ABD] + [ACD] + [BCE] + [BDE] + [CDE].
- In $K(r_2)$, [BCD] is added, while the 3-simplices [ABCD] and [BCDE] are added in $K(r_3)$ and $K(r_4)$, respectively.
- In this filtration, there is a persistent pair (r_1, r_4), where [ABC] is the positive simplex and [BCDE] is the negative simplex. According to the definition, the persistent volume must contain [BCDE], and thus it can be either [BCDE] or [ABCD] + [BCDE].
- However, ∂[BCDE] = [CDE] + [BDE] + [BCE] + [BCD], and for [BCD]∈ $K(r_4) - K(r_1)$,

  $[BCD]^*$ (∂[BCDE]) = $[BCD]^*$([BCD]) = 1 ≠ 0.

  Thus [BCDE] is not a persistent volume for (r_1, r_4).
- On the other hand, we can similarly check that [ABCD] + [BCDE] is a persistent volume for (r_1, r_4).

  And [ABC] + [ABD] + [ACD] + [BCE] + [BDE] + [CDE] is the corresponding volume-optimal 2-cycle.

Removing non-manifold vertices and edges

- When a volume-optimal cycle is obtained, since the original point cloud may have noise, it may contain non-manifold vertices and edges.
- Since these vertices and edges will lead to wrong results of Loop subdivision, it is necessary to detect non-manifold vertices and edges and remove them.

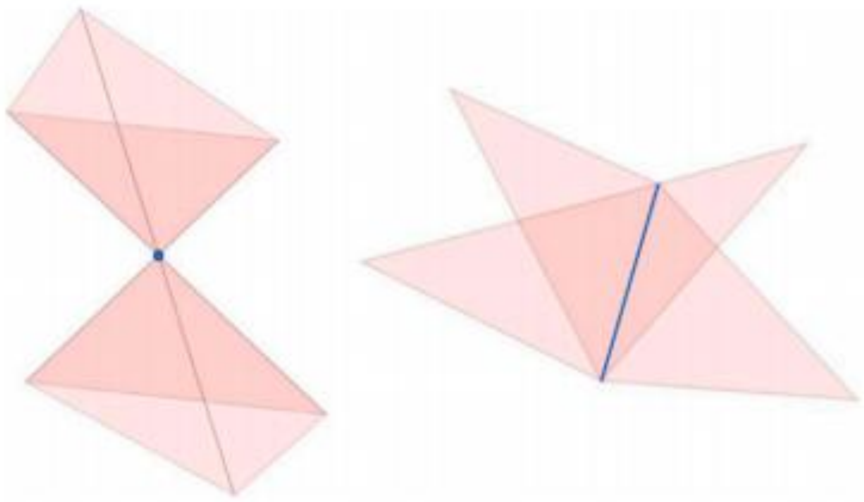


Fig. 7. Left: an example of a non-manifold vertex. Right: an example of a non-manifold edge.

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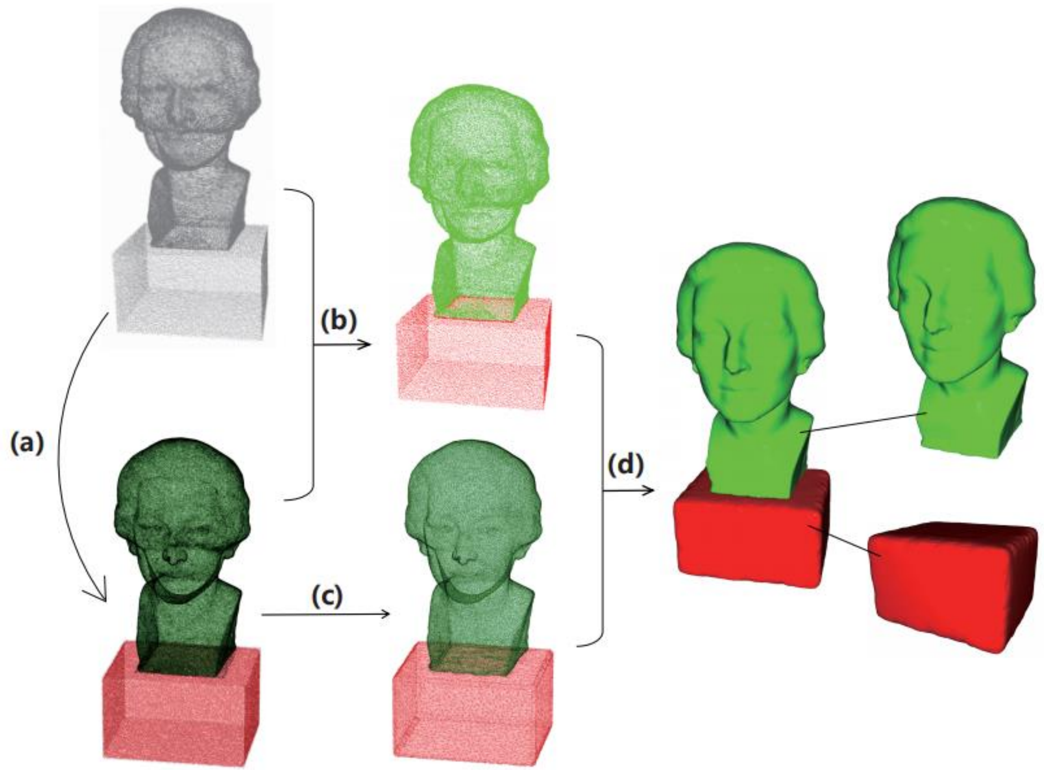
Loop Subdivision Fitting and Optimization

- After getting representative 2-cycles and removing singular vertices and edges, we use QEM mesh simplification method to simplify the number of vertices to about 20%.
- Then, these 2-cycles are regarded as the control meshes, and we use the Loop subdivision fitting and least squares progressive iterative approximation (LSPIA) techniques to obtain the reconstructed model.
- These geometric processing techniques can make the results better fit the noisy point cloud and lead to high-quality reconstructed models.



Algorithm 1 Model Reconstruction Algorithm

Input: A point cloud P representing a model.
1: Construct the alpha filtration from P, then calculate persistent homology and the 2-PD.
2: Cluster the points in 2-PD and derive the significant points.
3: Compute persistent volume and volume-optimal cycle for each significant point in 2-PD.
4: Identify all non-manifold vertices and edges for each volume-optimal cycle, then remove 3-simplices in the corresponding persistent volume that connect with non-manifold vertices or edges. Compute the boundary of the final persistent volume as the new representative 2-cycle.
5: Compute neighboring points (Eq (4)) for each 2-cycle.
6: Reduce each 2-cycle to a simpler mesh using the QEM method, then use the reduced 2-cycle as the initial control mesh and apply Loop subdivision to generate refined meshes.
7: Apply LSPIA to optimize each obtained mesh surface with its neighboring points until the stopping criterion is reached.
Return: The reconstructed surfaces of the components of the model.

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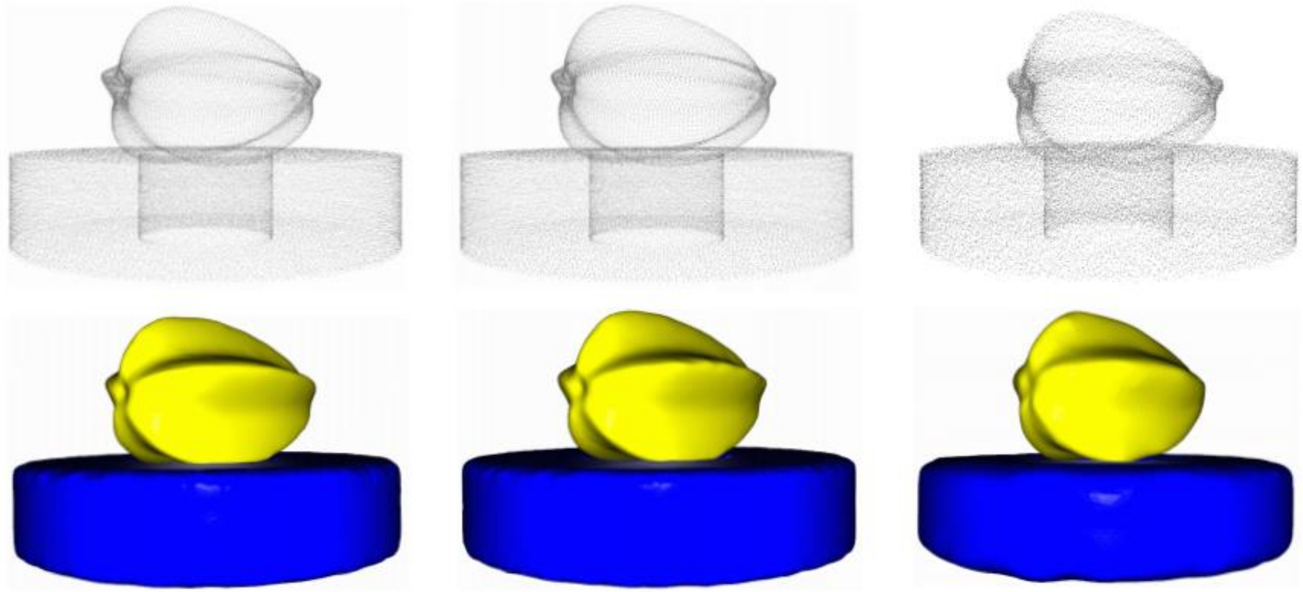
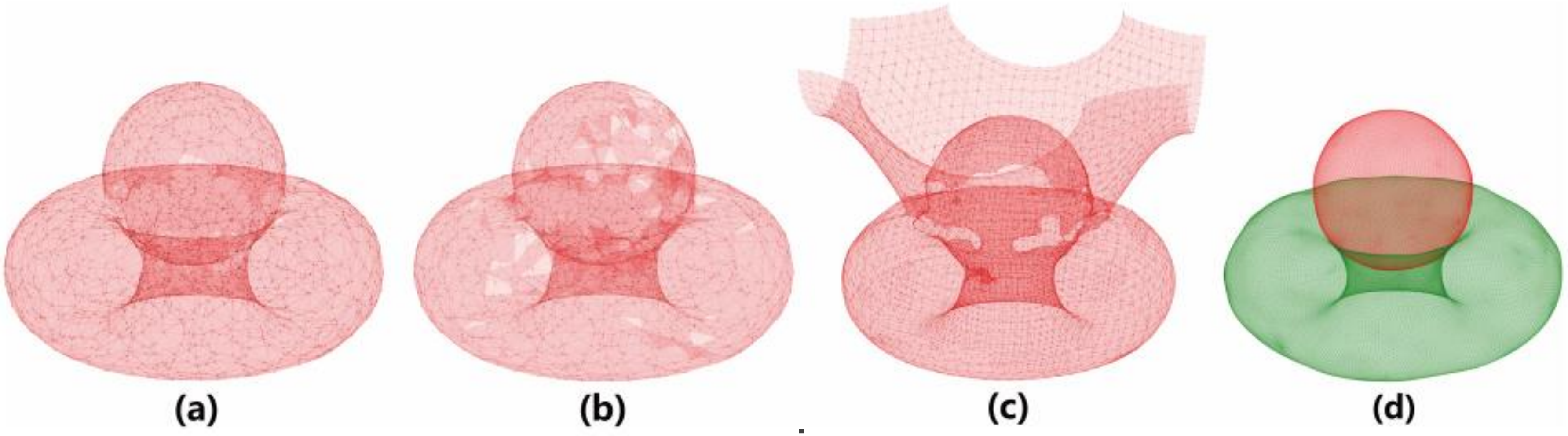
Robustness to Noise and Comparison



robustness to noise



comparisons

Results of Reconstructing CAD Models

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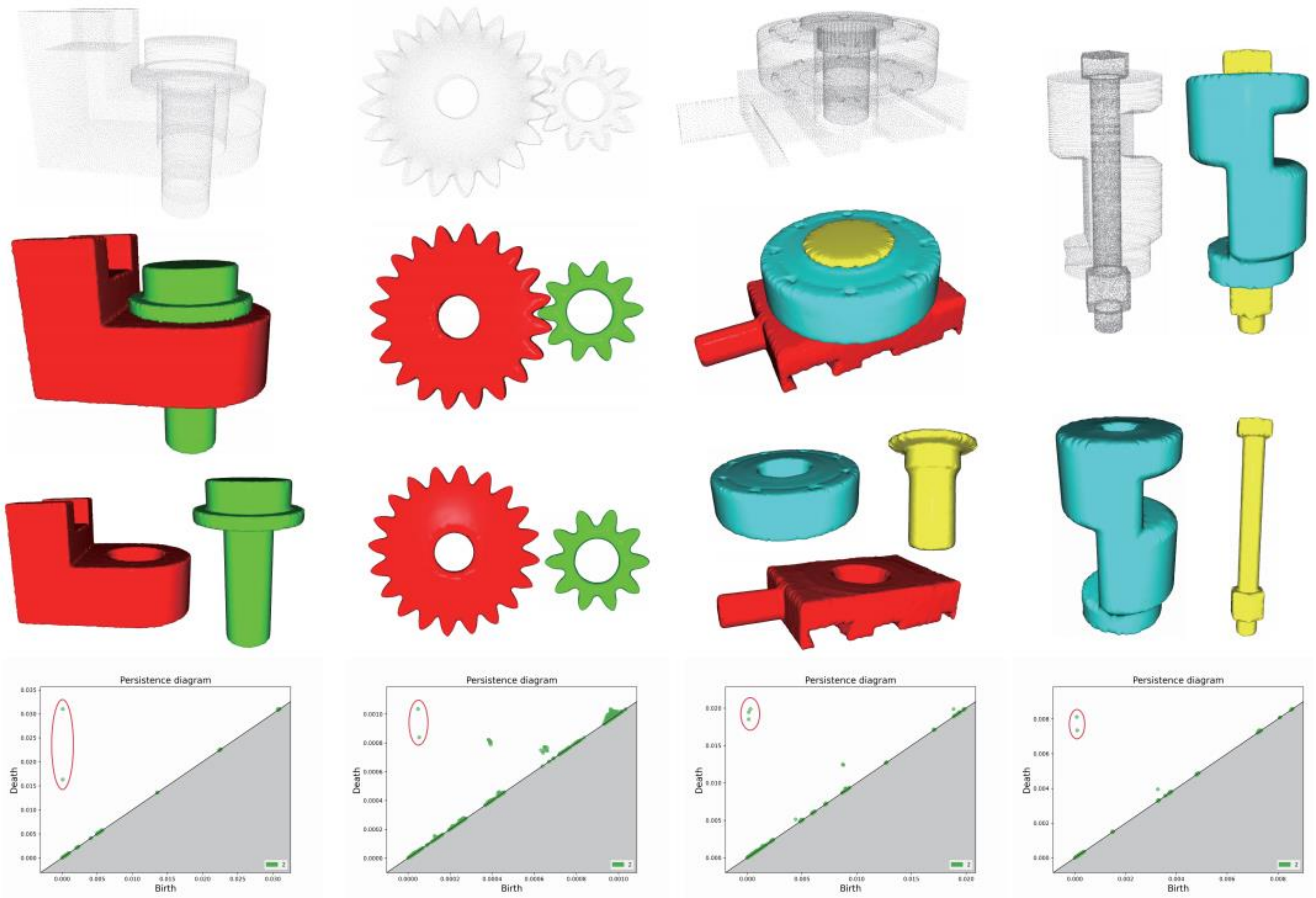

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Results of Reconstructing Real-world Object Models

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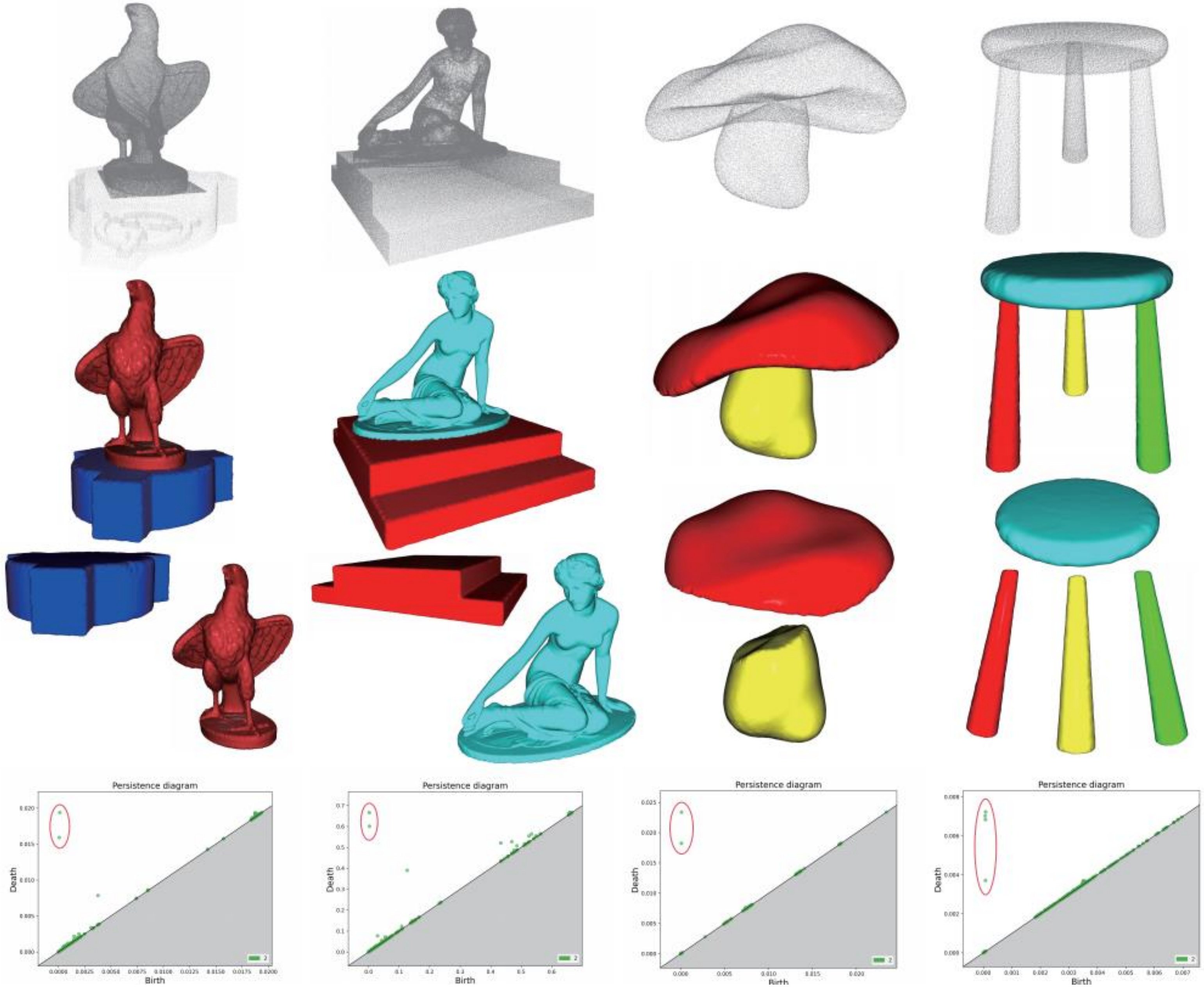

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Results

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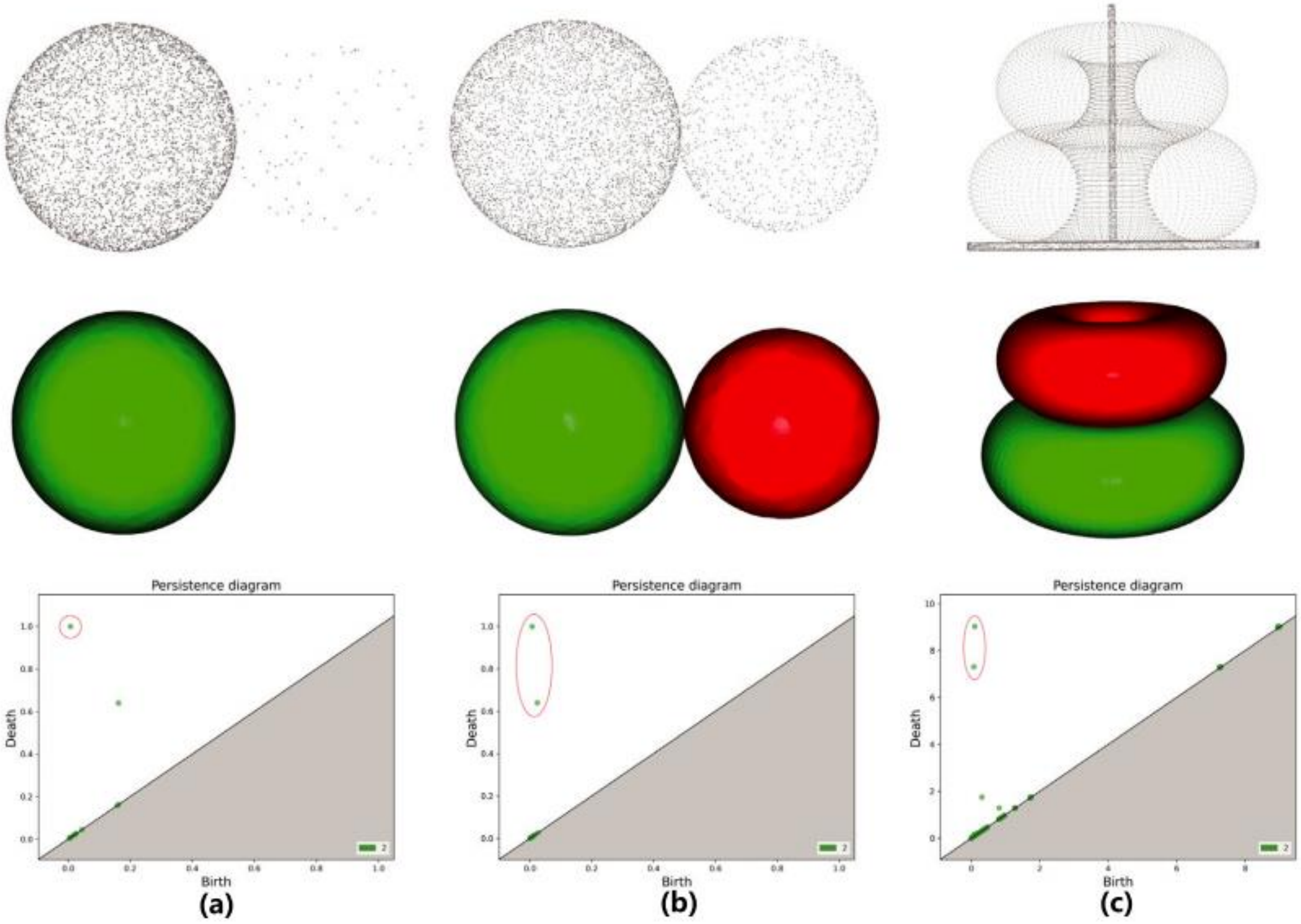
➢ The reconstruction results can not only reconstruct all components (correct topology), but can also achieve small errors (fine geometry).

Table 1: Statistics on point clouds and reconstructed results.

Point Cloud	Number of Points	Number of Control Vertices	Number of Surfaces	RMSE	Time of Computing Topology	Time of Subdivision Fitting
Fig. 1	84157	31314	5	3.277×10^{-4}	1506.5s	463.7s
Fig. 10 (a)	34173	8332	2	1.184×10^{-4}	239.5s	520.0s
Fig. 10 (b)	38644	3452	2	1.476×10^{-5}	195.8s	466.0s
Fig. 10 (c)	45618	14870	3	3.339×10^{-4}	404.0s	684.2s
Fig. 10 (d)	50146	12163	2	1.649×10^{-4}	351.6s	925.8s
Fig. 11 (a)	300869	51692	2	2.246×10^{-4}	22122.0s	1384.9s
Fig. 11 (b)	598144	67050	2	7.178×10^{-4}	16987.1s	2822.7s
Fig. 11 (c)	87656	9986	2	1.828×10^{-4}	1454.9s	523.1s
Fig. 11 (d)	73312	6943	4	1.178×10^{-4}	1591.8s	421.2s

Future works



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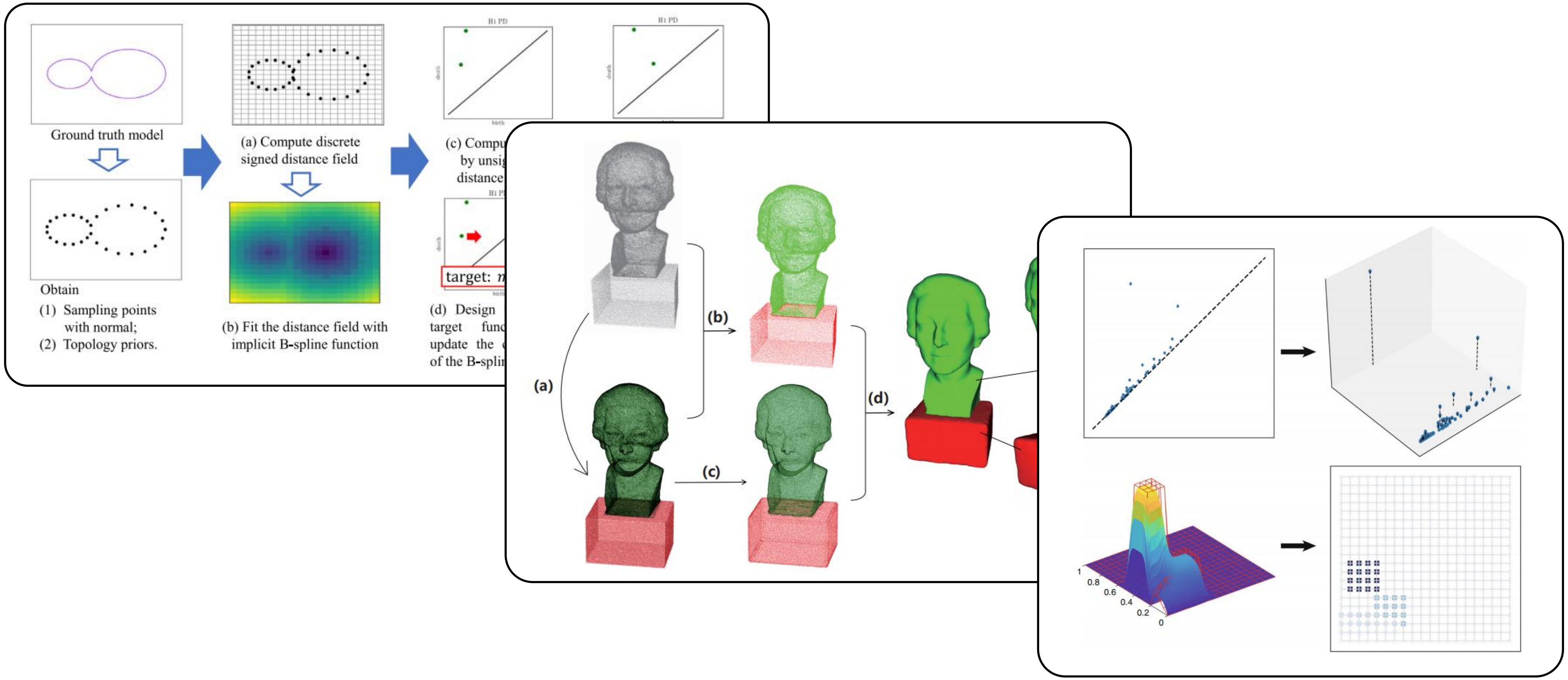
Conclusions

- We develop a method to reconstruct a whole model. The number of reconstructed closed surfaces is determined by topological understanding using persistent homology.
- The initial control meshes for surface reconstruction are represented by the representative cycles of persistent homology.
- The LSPIA method for subdivision surfaces is employed to generate reconstructed surfaces that better approximate the given point clouds.

Computer-Aided Topological Design

- Computational Geometry ⟶ Computer-Aided Geometric Design (CAGD)
- Computational Topology ⟶ Computer-Aided Topological Design (CATD)



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Computer-Aided Topological Design

- **Topology Understanding**: computational topology offers a mathematical framework to understand and verify the topological features of geometric models.
- **Topology Control**: computational topology enables designers to generate models that not only satisfy geometric requirements but also possess precisely controlled topological properties.
- With the development of TDA and TDL, CATD will be more important in CAD realm.

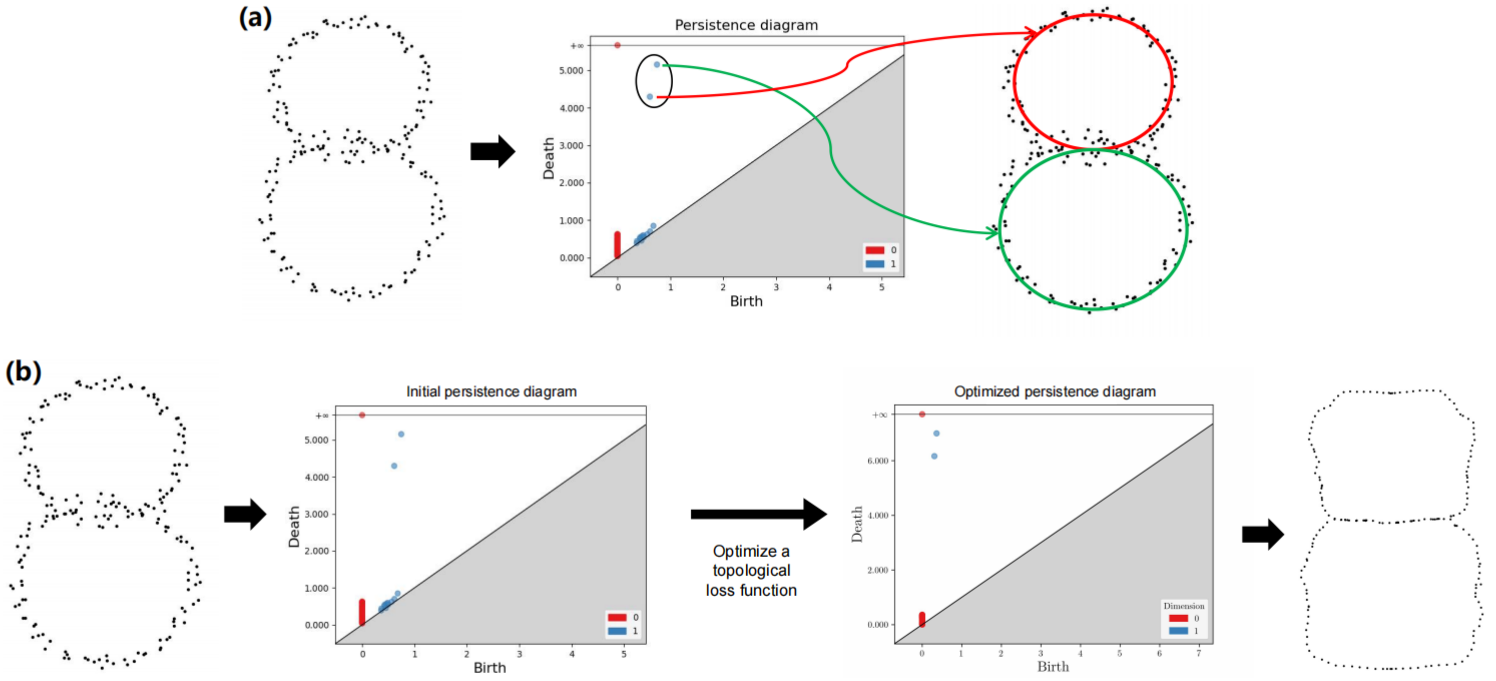

Fig. 1. An illustration of the pipelines of topology understanding and topology control. (a) Topology understanding of a point cloud by identifying significant points in persistence diagrams and detecting corresponding topological features. (b) Topology control of the shape of a point cloud by designing and optimizing a topological loss function constructed by the persistence diagram.

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Thank you!