

Topological combined machine learning for consonant recognition



Yifei Zhu

Southern University of Science and Technology

2023.11.26

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Can we see the sound of a human speech?

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Periodic phenomena: a motivating example

Let $\mathbb{T}^2 = (\mathbb{R}/\mathbb{Z})^2$ be the 2D torus. Consider the dynamical system given by

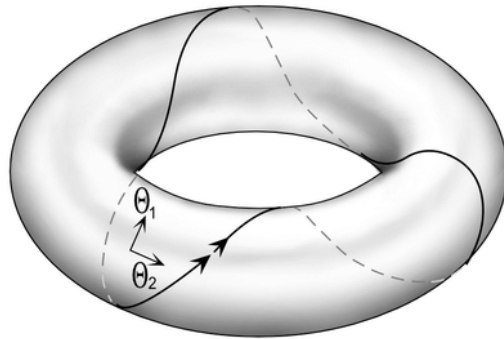
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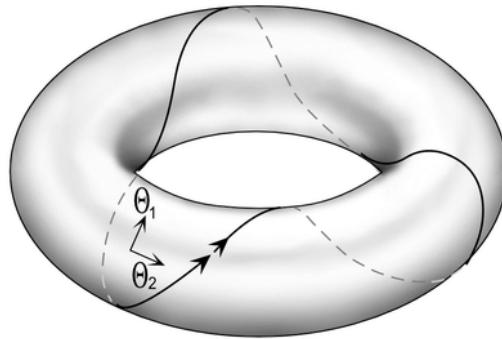


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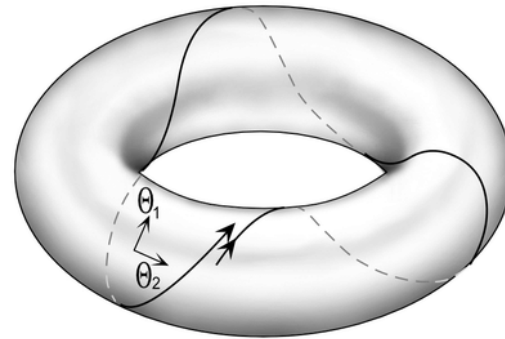
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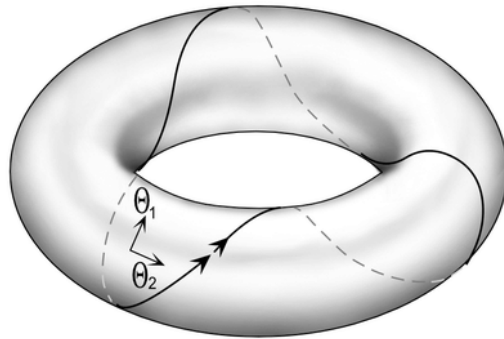
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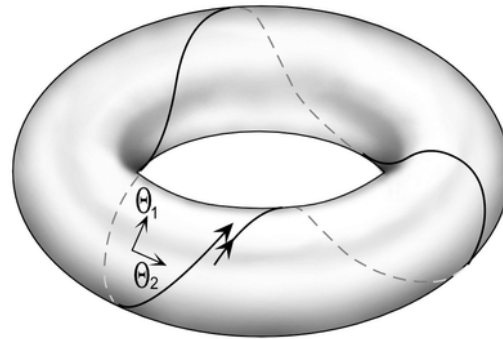
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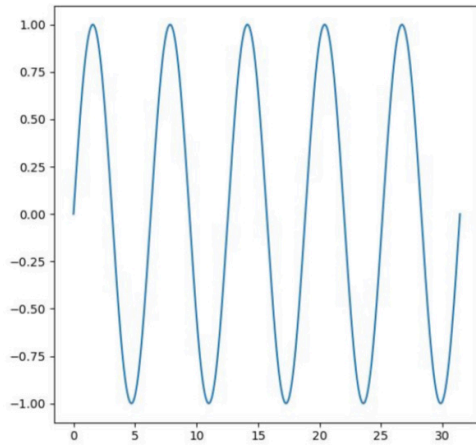
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From time series to topological shapes

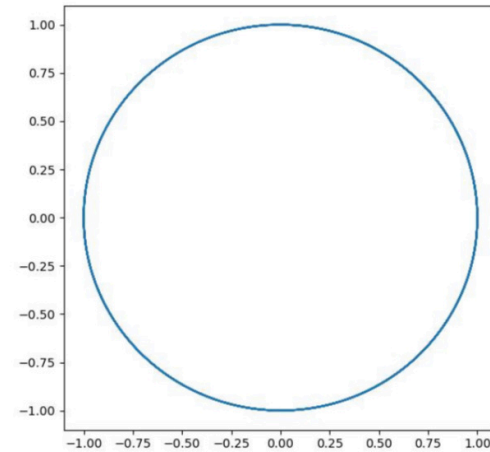
Most periodic time series can be realized by a **topological circle S^1** embedded in a Euclidean space of higher dimension.

Ideas of topological data analysis (TDA)

The topological type (more precisely, homotopy type) is **robust** against perturbations.

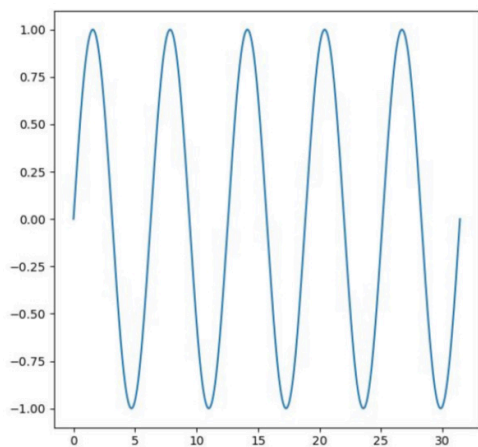


$$y = \sin x$$

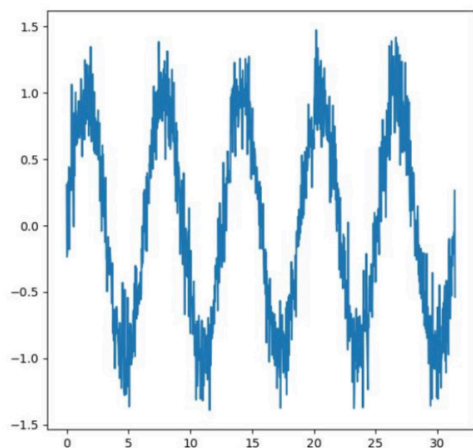
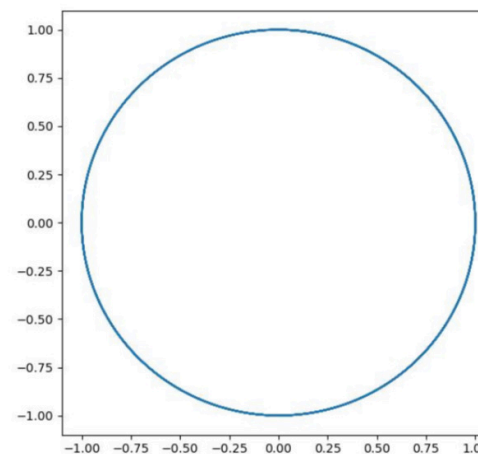


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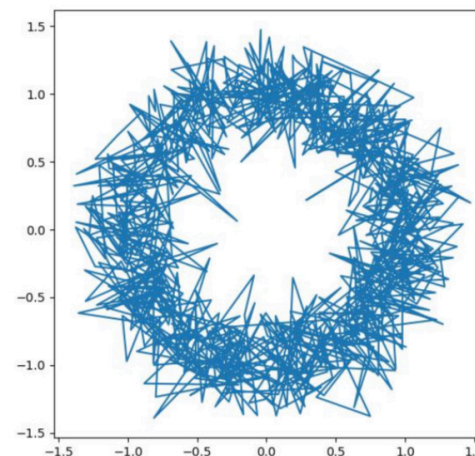
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$$y = \sin x + \epsilon(x)$$



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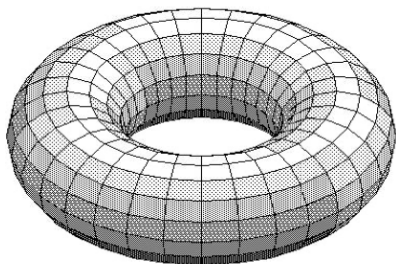
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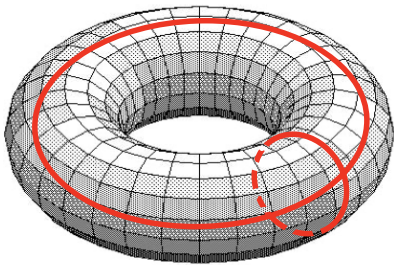


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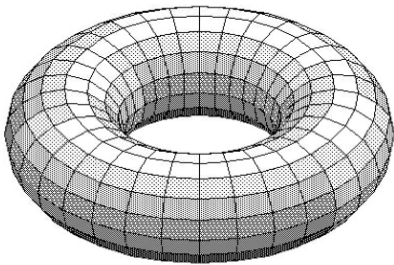


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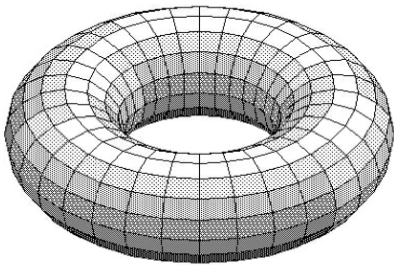


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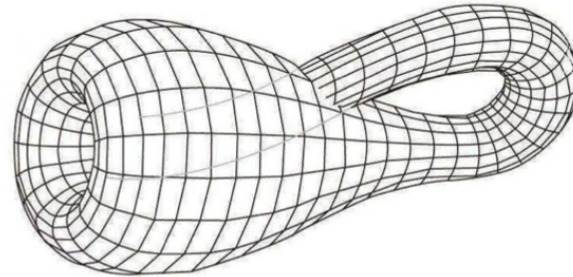
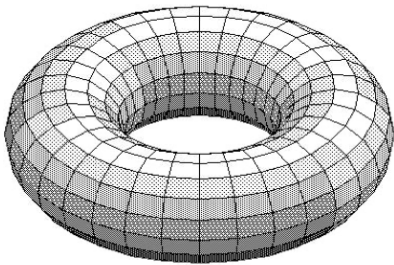


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$$H_k(\text{Klein bottle}) = \begin{cases} \mathbb{Z} & k = 0 \\ \mathbb{Z} \oplus \mathbb{Z}/2 & k = 1 \\ 0 & k = 2 \\ 0 & k > 2 \end{cases}$$

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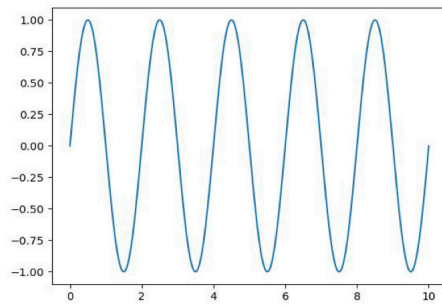
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Topological time series analysis

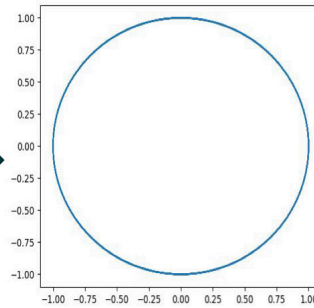
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← realization



← computation

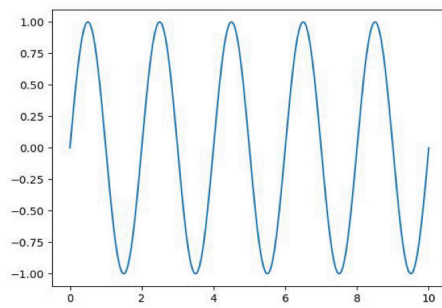
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Topological time series analysis

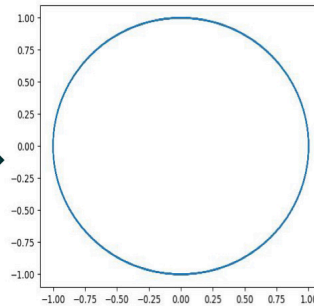
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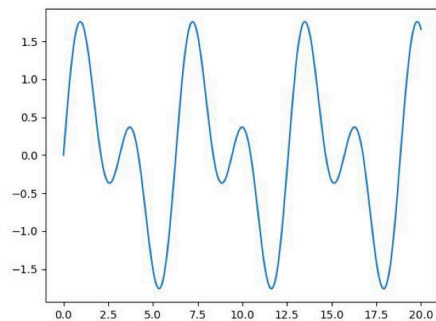


realization

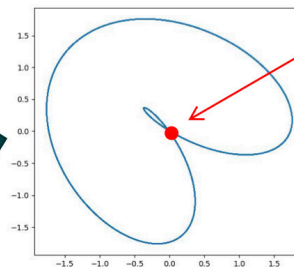


computation

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not an embedding



self-intersection

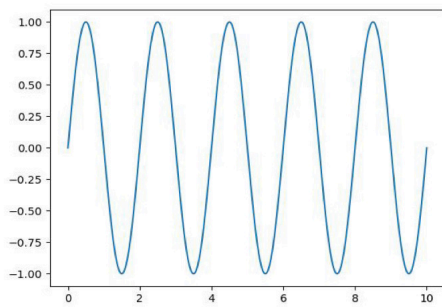
2D

Topological time series analysis

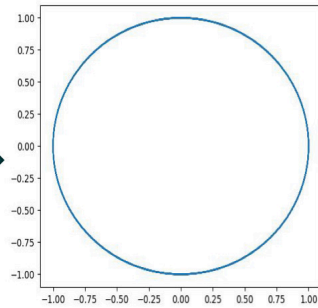
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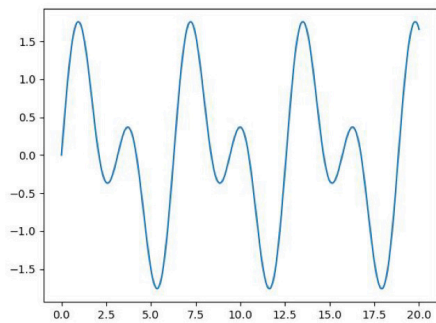


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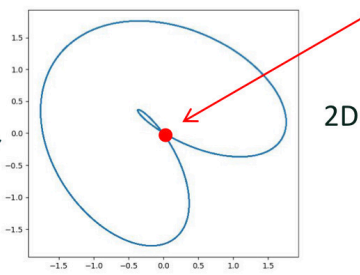


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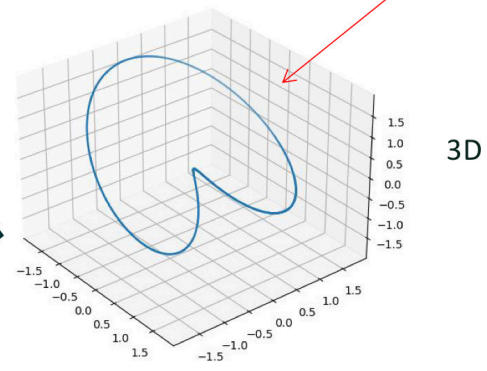
not an embedding



self-intersection

a topological circle

an embedding (preserves topological information)



An application: detection of wheeze in medical science (pulmonology)

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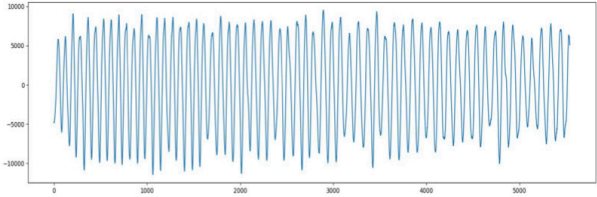


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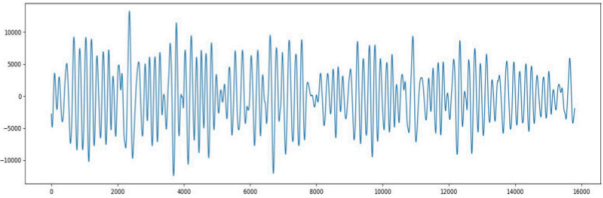
As a warm-up, our research group (Siheng Yi) has reproduced their results using the original data and open-source TDA programming package.

An application: detection of wheeze in medical science (pulmonology)

wheeze



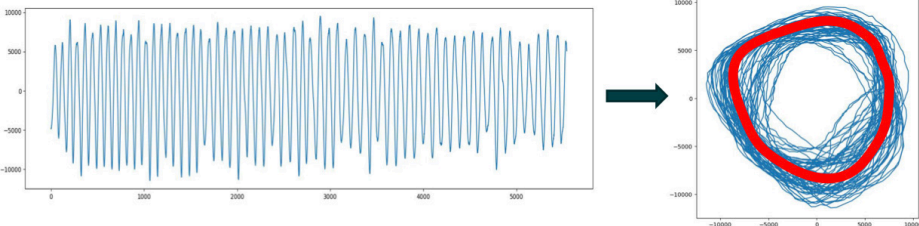
normal



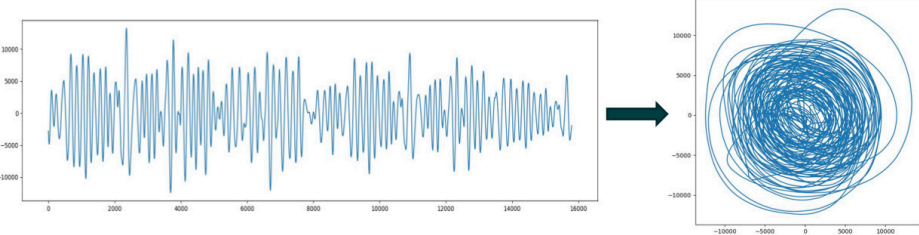
Original sound signals

An application: detection of wheeze in medical science (pulmonology)

wheeze



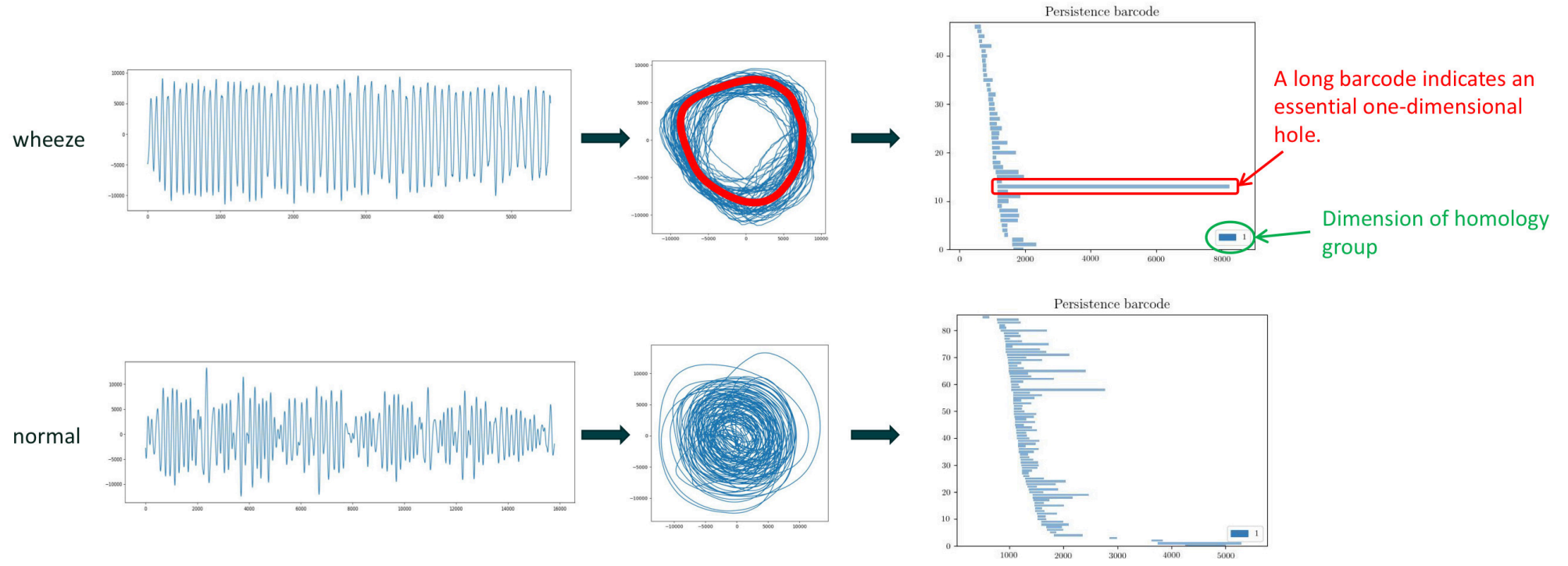
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Original sound signals

Realized topological shapes embedded in 2D Euclidean space

An application: detection of wheeze in medical science (pulmonology)



A long barcode indicates an essential one-dimensional hole.

Dimension of homology group

Original sound signals

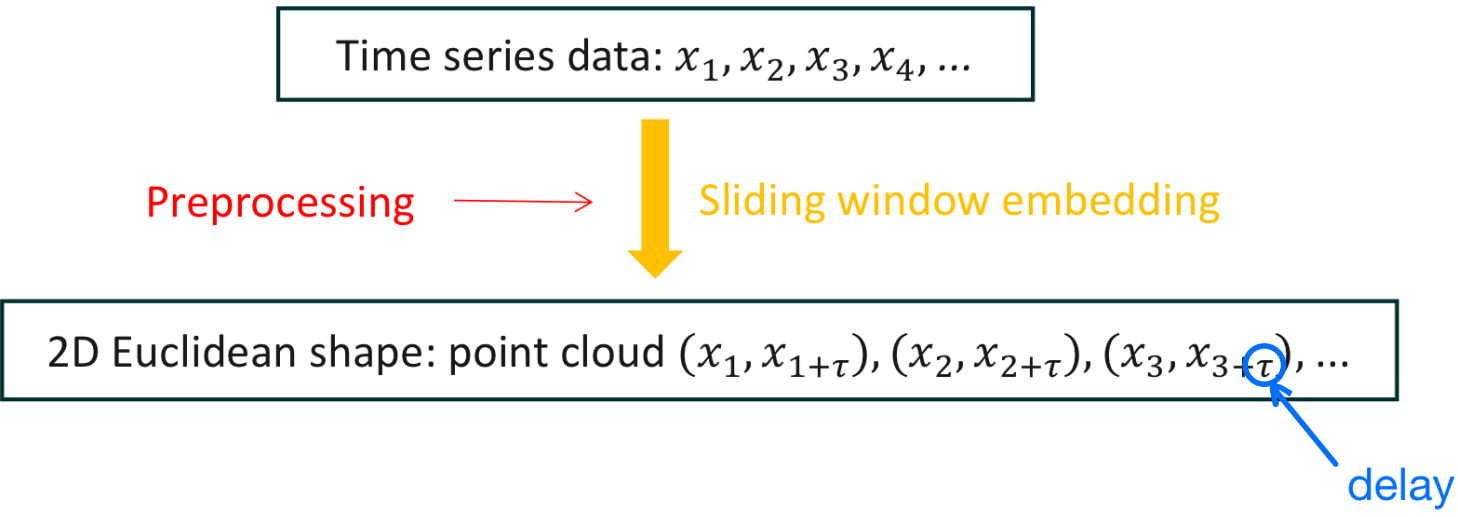
Realized topological shapes embedded in 2D Euclidean space

“Persistence barcodes” as representations of the algebraic invariant (1D homology group)

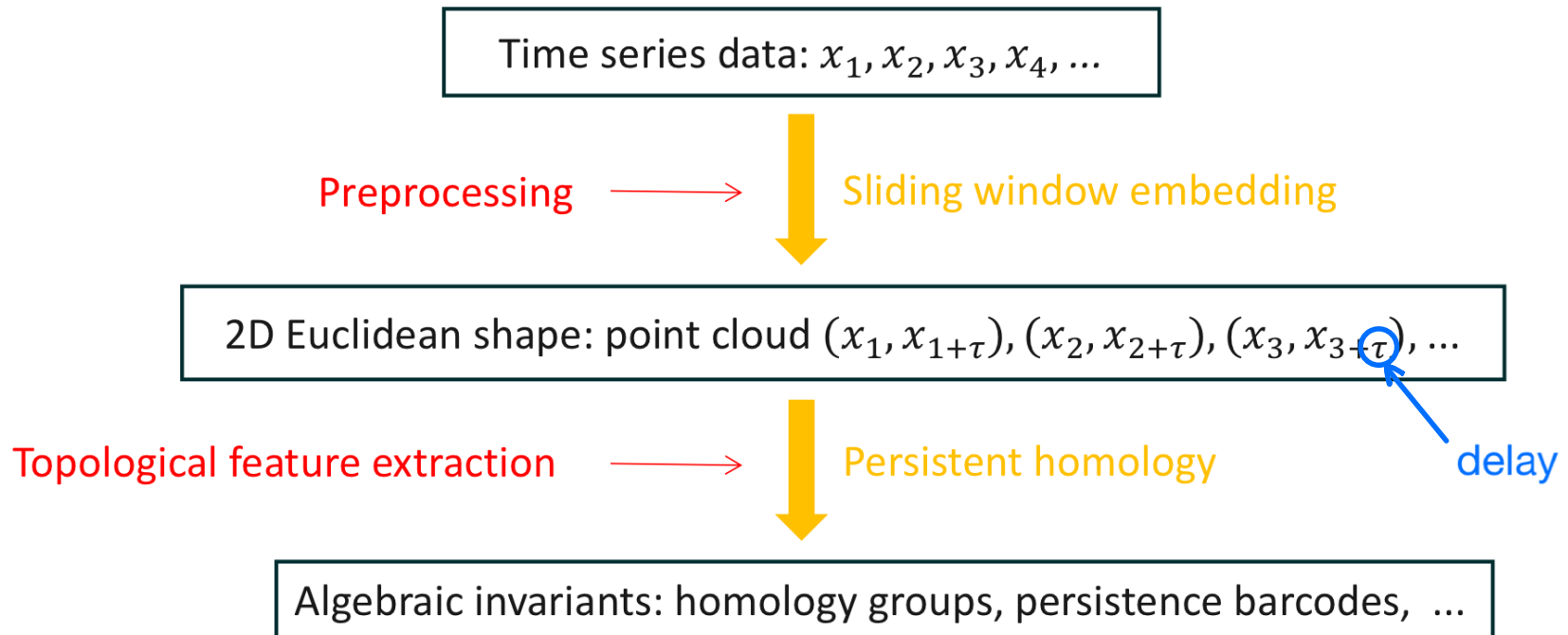
A pipeline for topological time series analysis

Time series data: $x_1, x_2, x_3, x_4, \dots$

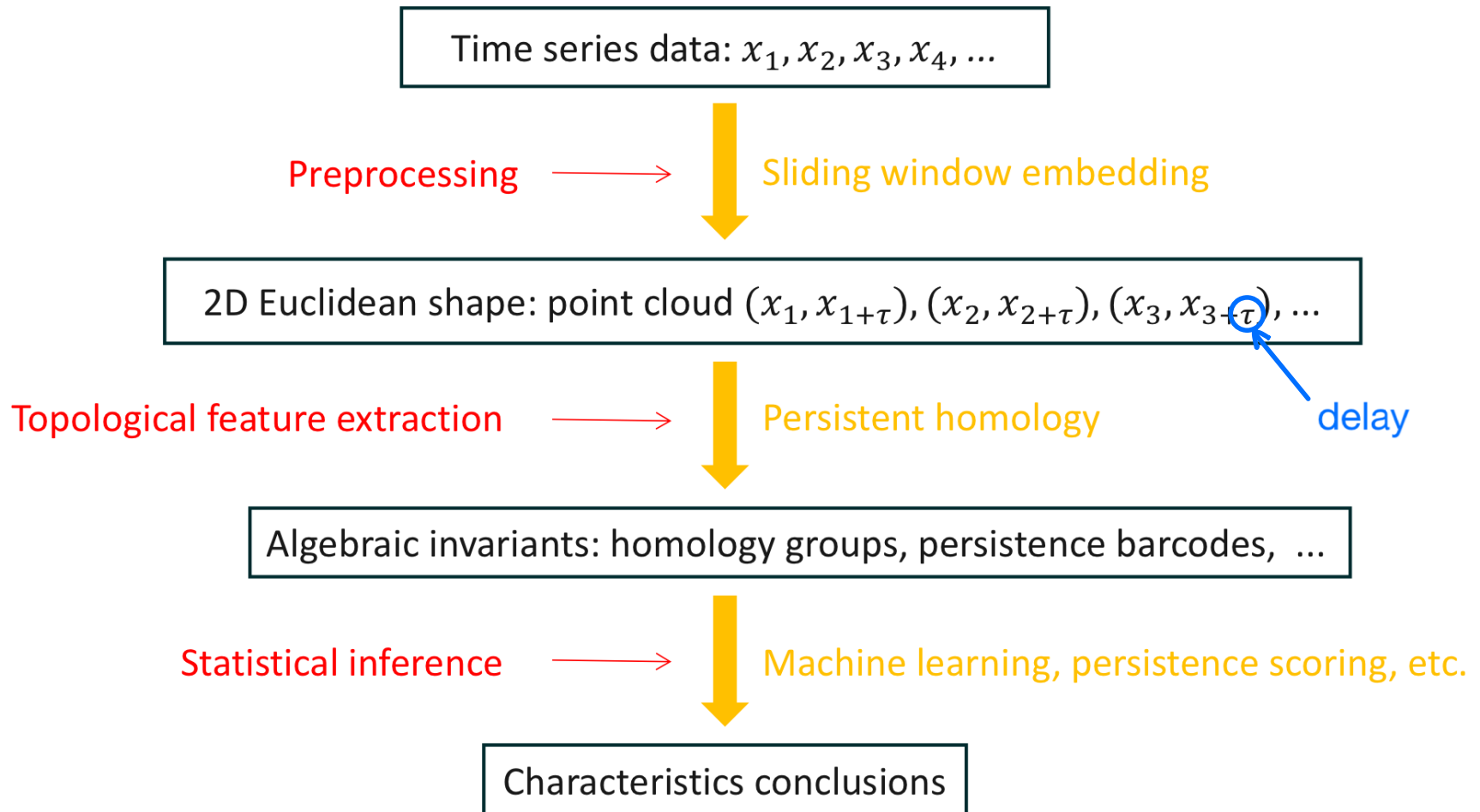
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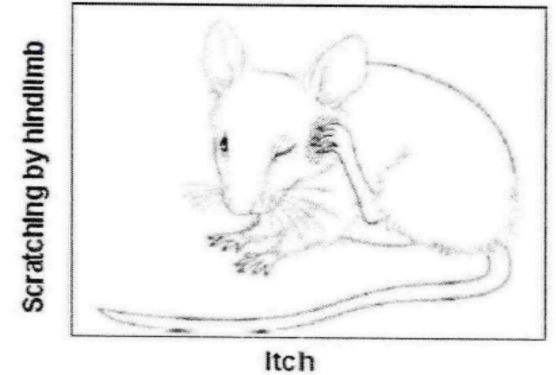


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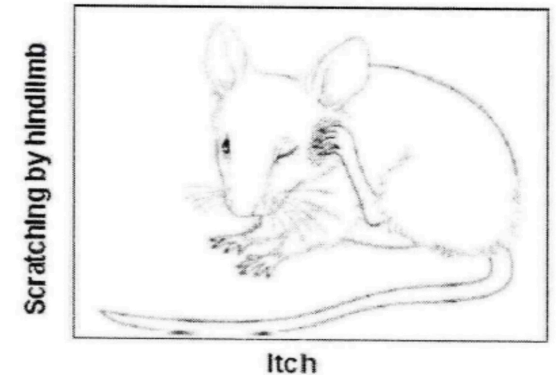
Application I: detection of mouse scratching behavior

Joint with the biomedical engineering group led by Fangyi Chen and the data science group led by Zhen Zhang, both at SUSTech, we applied topological methods to the problem of **automated** and **real-time** detection of mouse scratching behavior, with motivations from **pharmacology**.

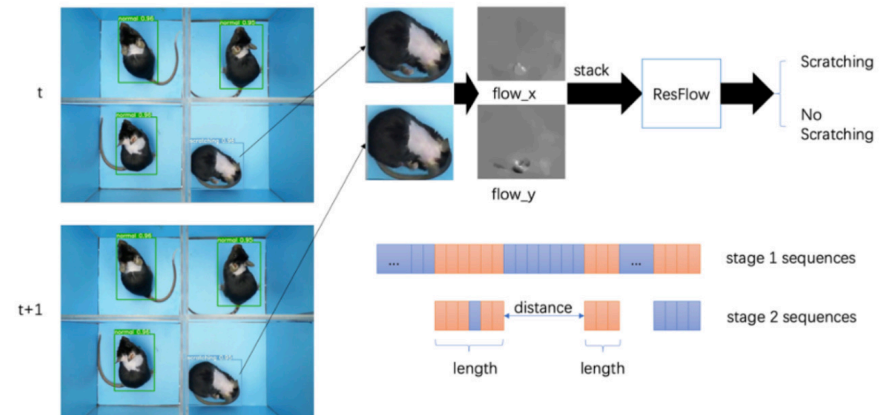


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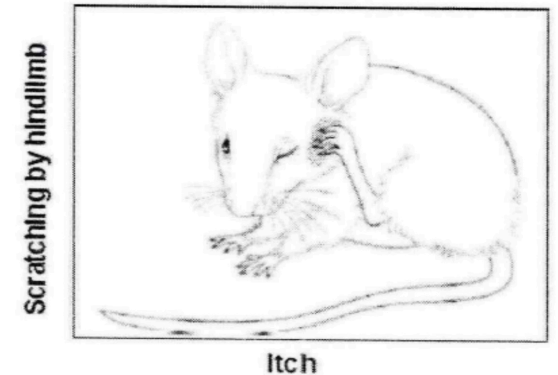


Prior to our group's involvement, machine learning via neural networks was applied with satisfactory accuracy (<https://yifeizhu.github.io/scratch.mp4>).

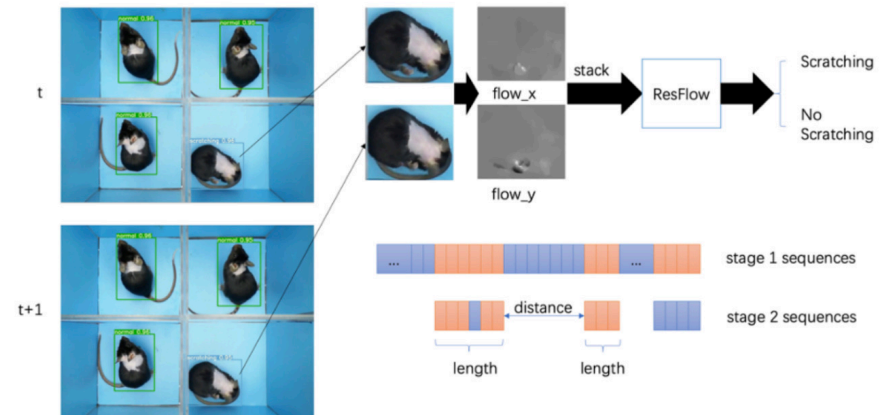


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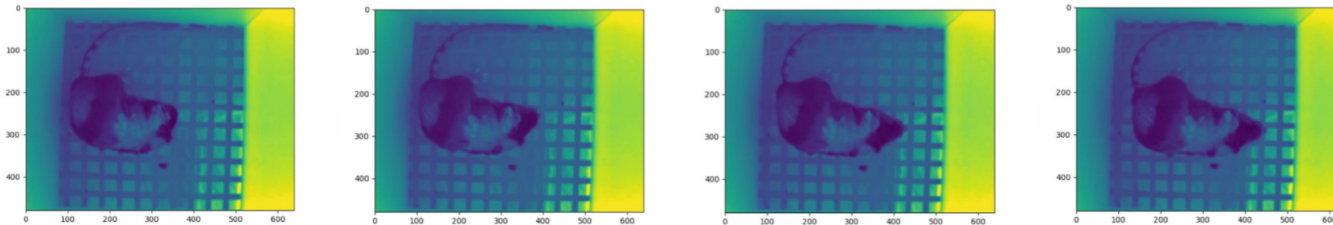
However, the learning process was **time consuming**, which is impractical for time-sensitive purposes and lab efficiency.

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We observed that the scratching behavior exhibits periodicity.

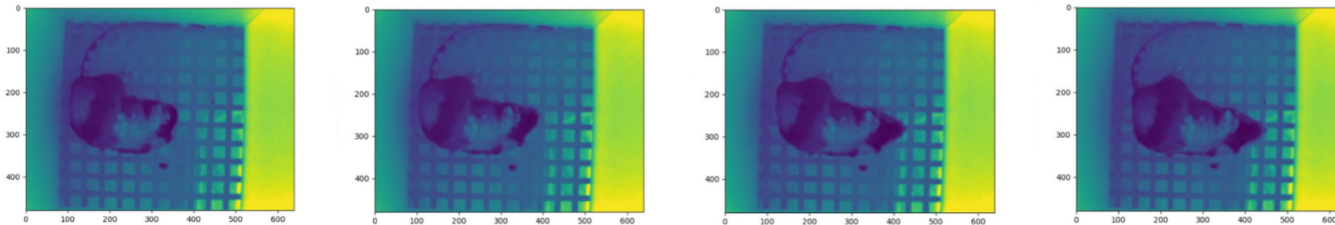
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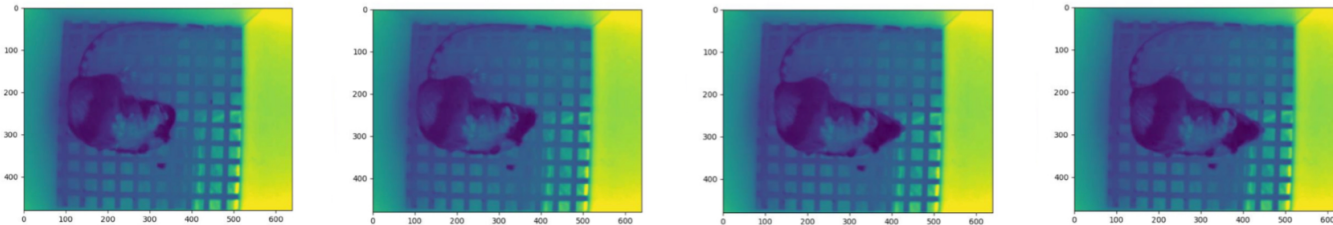


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Approach 1 **Sum up** all 460x640 pixels to extract a series of **1D data** which ignores differences caused by global movements. Too coarse?

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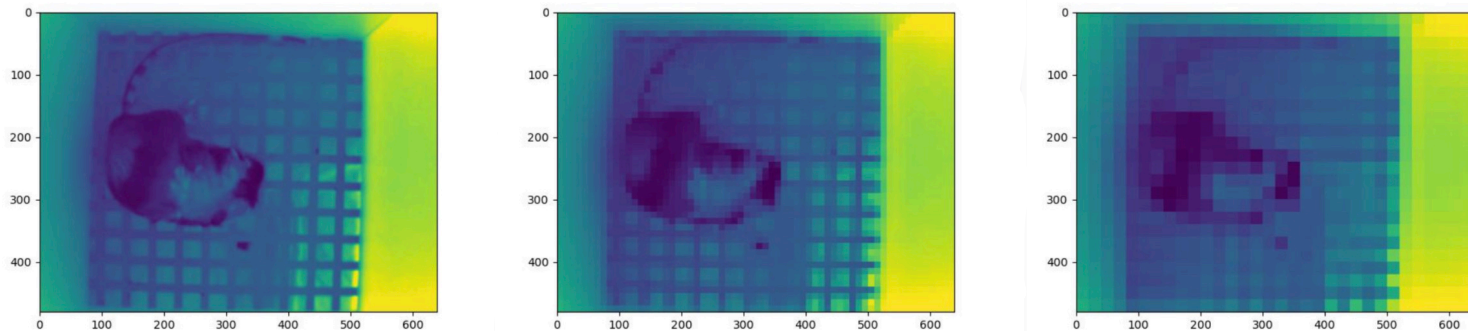
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Approach 2 Blur the images by **pooling**, and feed the topological pipeline with reduced **100-dimensional data**. Still too refined?

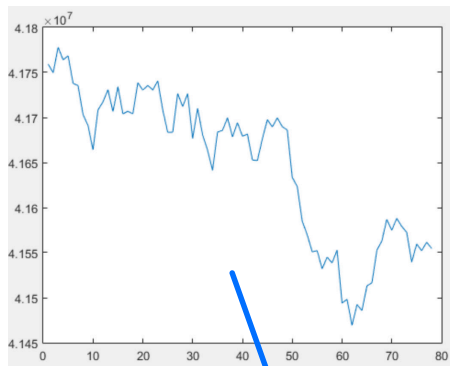


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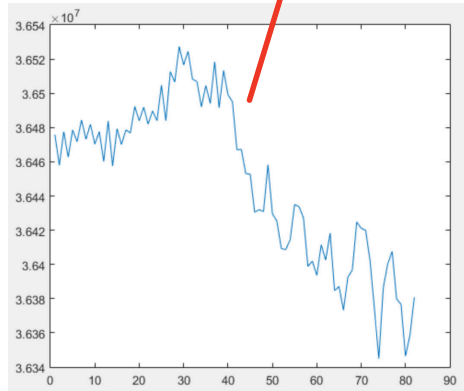
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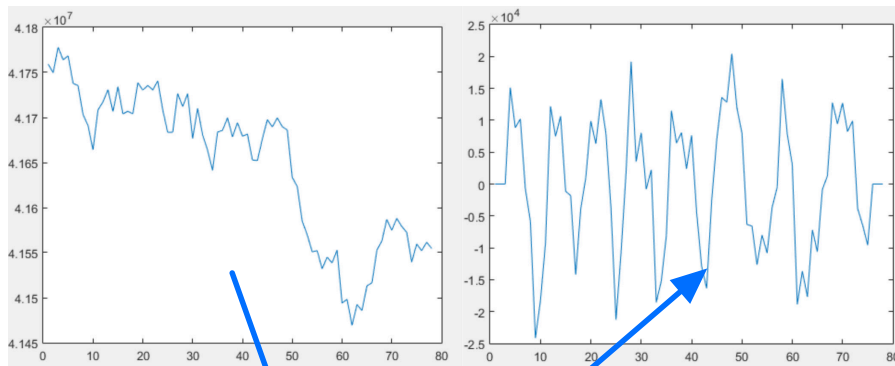


2 filtrations:
 $f' = f * K_3$
 $f'' = f' - f' * K_7$

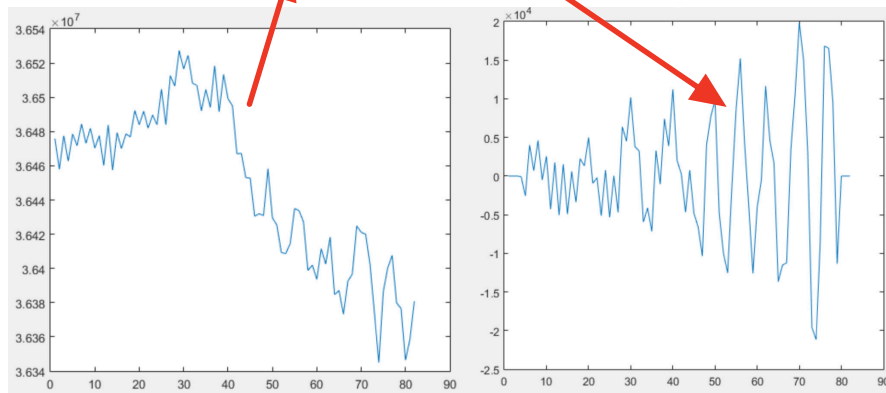


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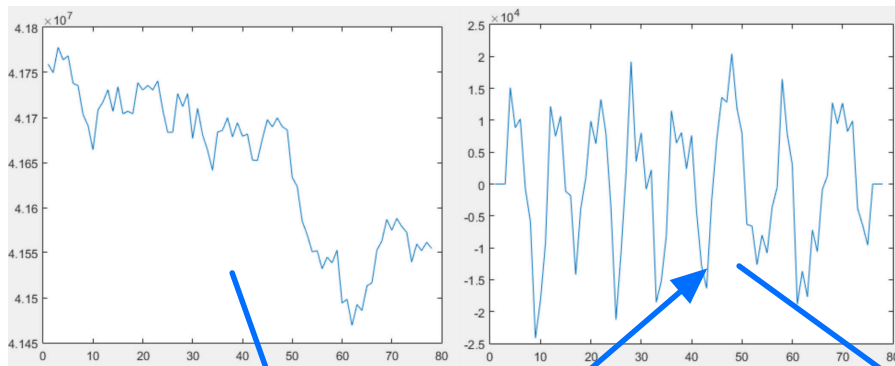


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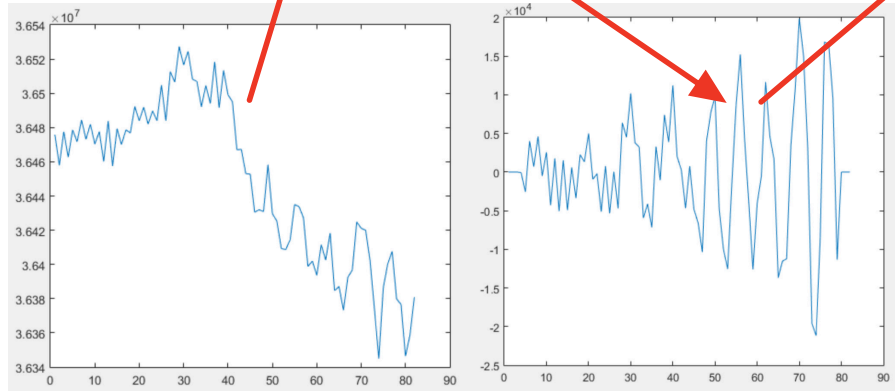
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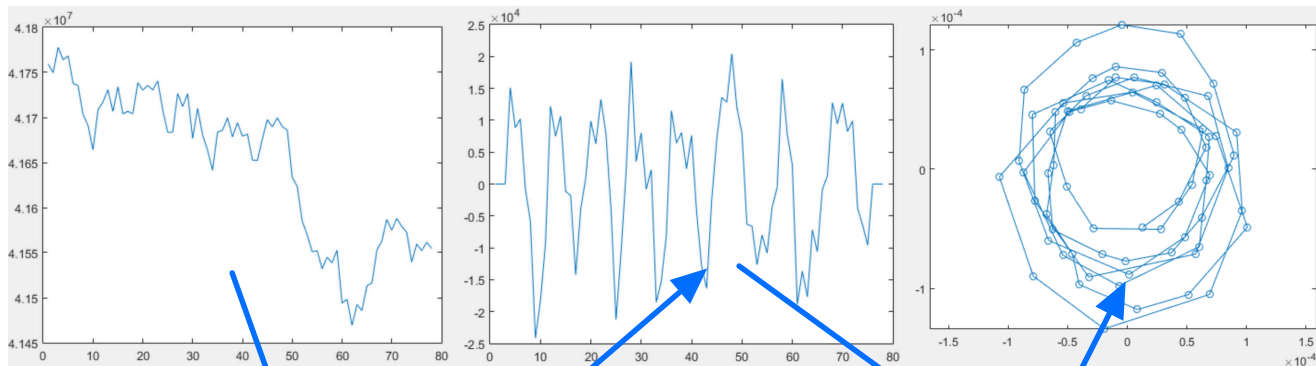
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Sliding window embedding (dim=6, delay=1)
then project to 2D



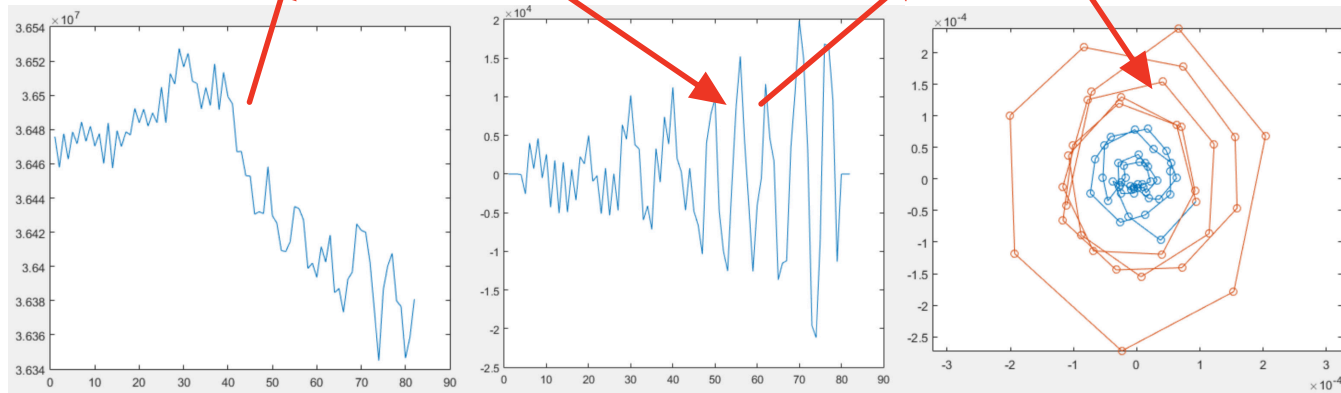
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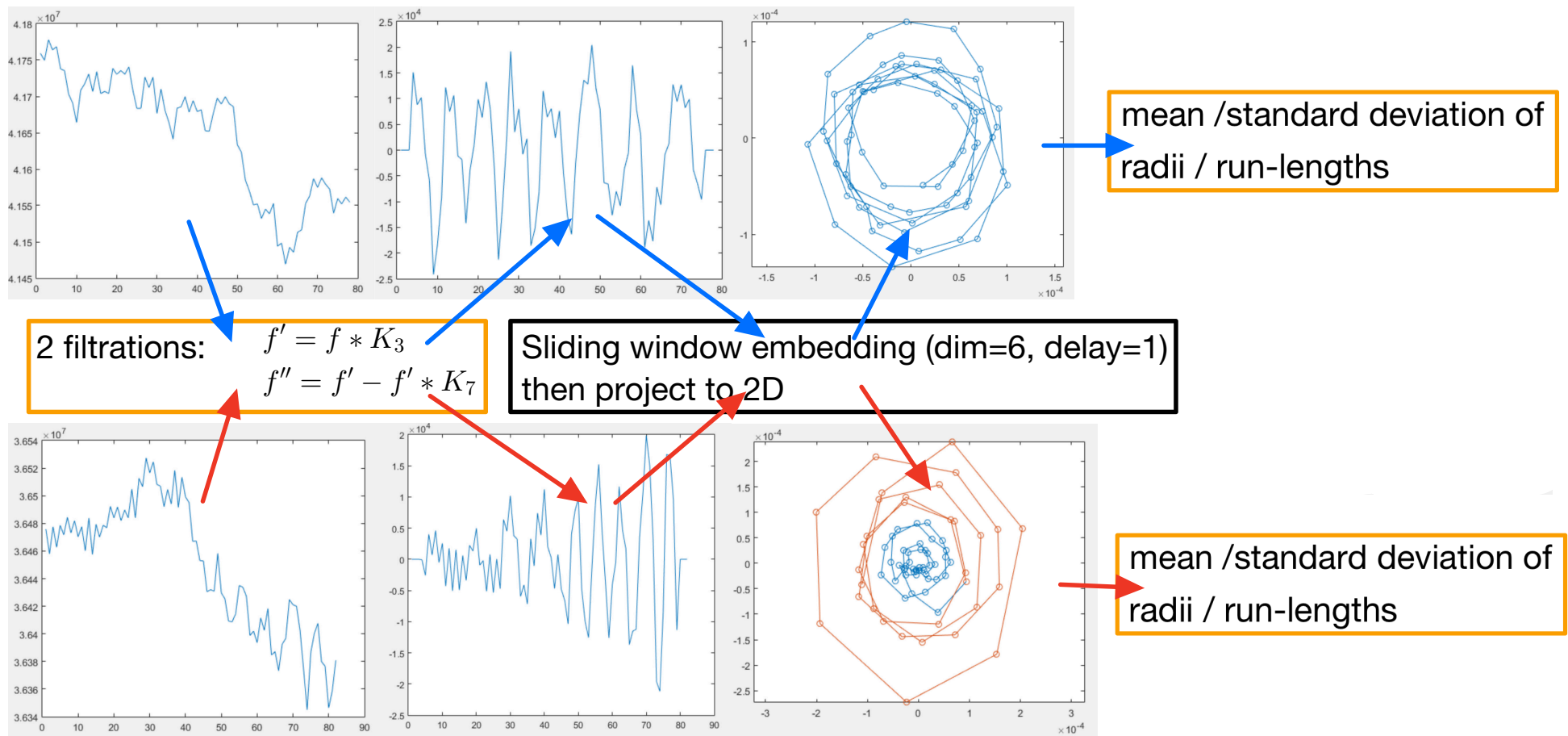
2 filtrations:
 $f' = f * K_3$
 $f'' = f' - f' * K_7$

Sliding window embedding (dim=6, delay=1)
then project to 2D



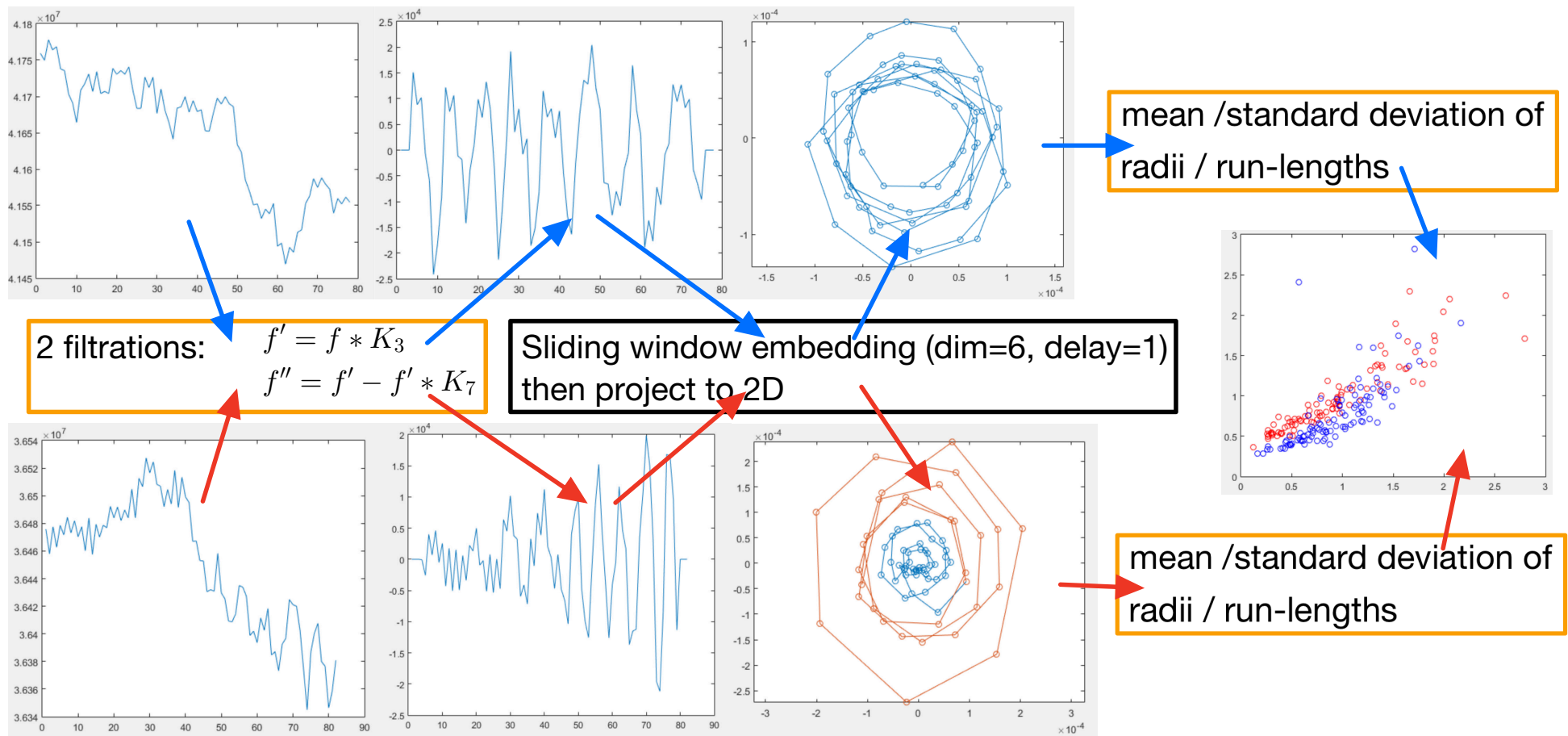
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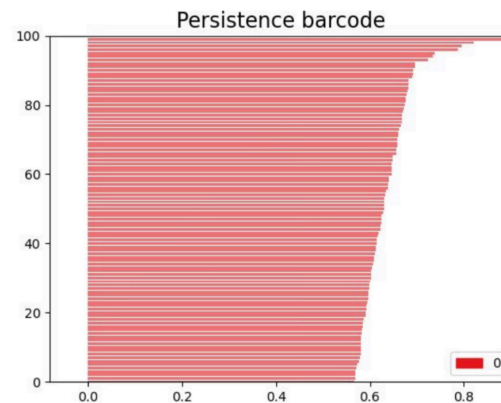
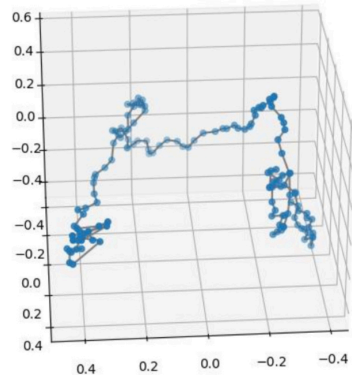
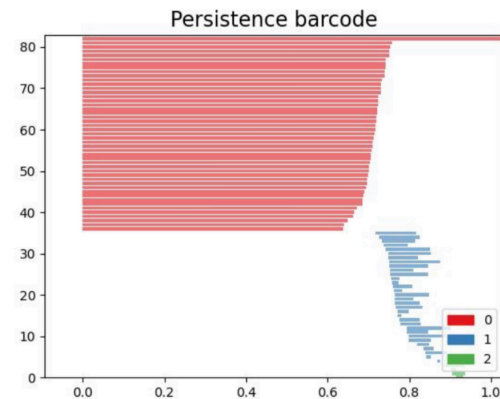
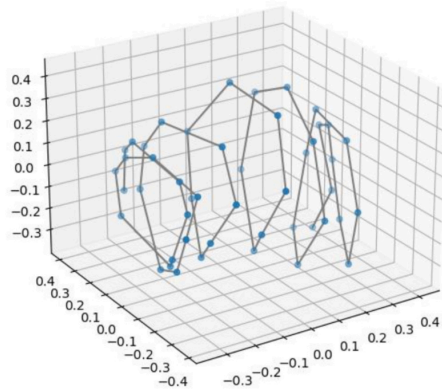
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Approach 2 (multi-dimensional data, Siheng Yi), combined with **persistent homology** and its representations, yielded recognizable characteristics but required considerable computational time.



Application II: classification of speech signals

Joint with Meng Yu of Tencent AI Lab, we applied topological methods to classify **voiced/voiceless** and **vowel/consonant speech** data, with motivations from industrial applications.

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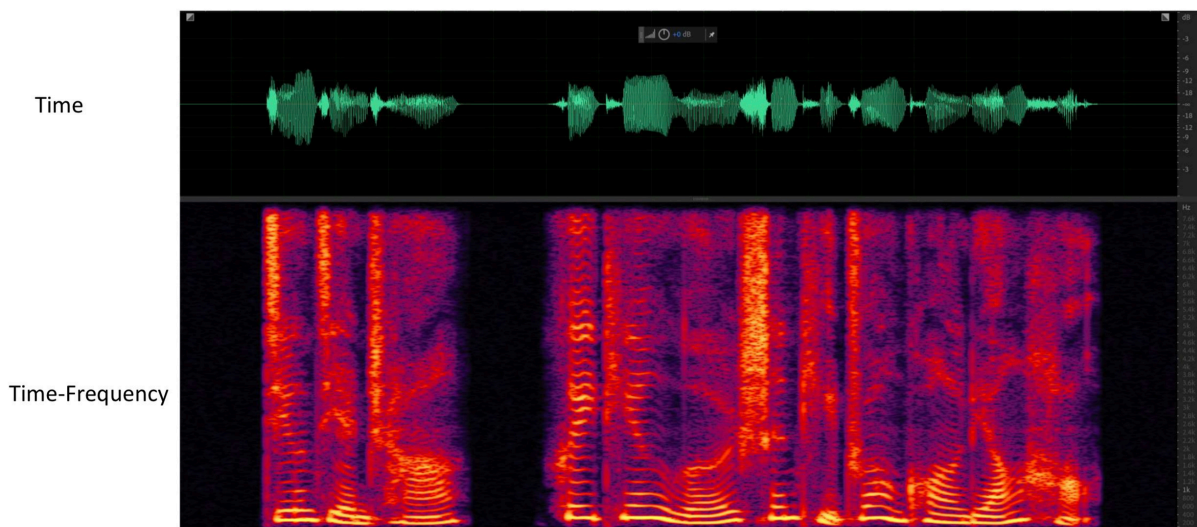
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Display of speech signals

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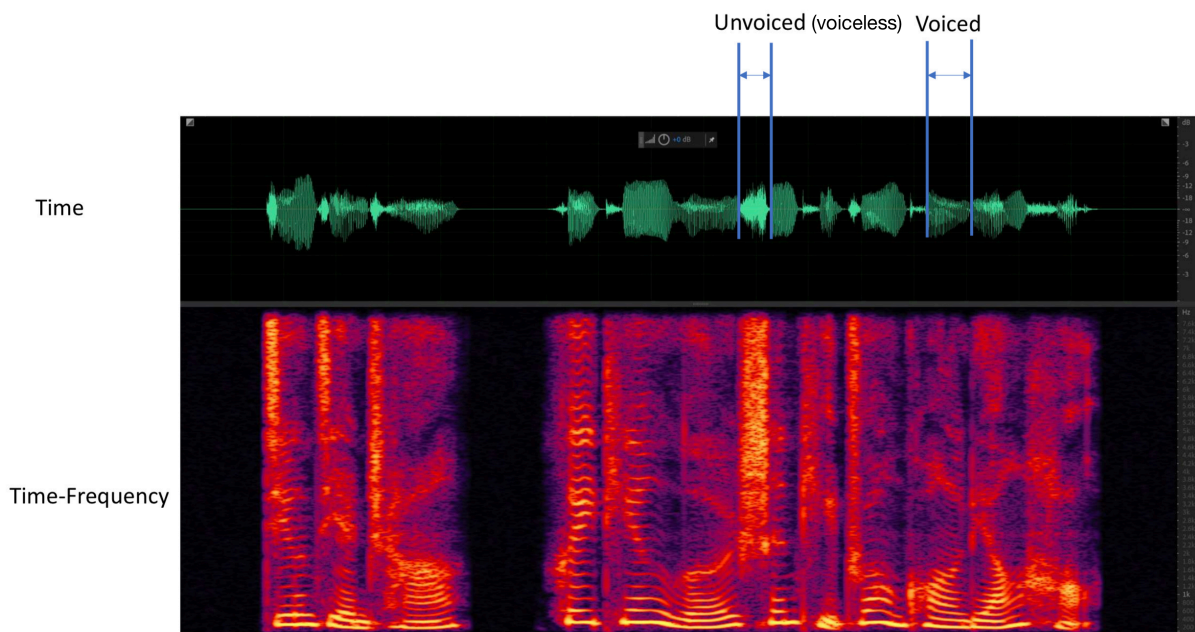
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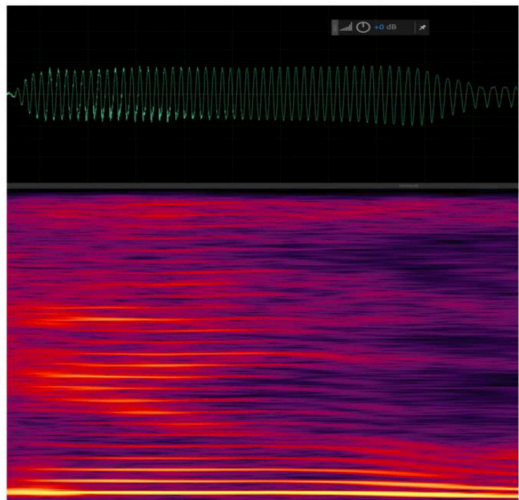


Application II: classification of speech signals

Voiced

Sinusoid in time domain

Harmonics in frequency domain



Time and Time-Frequency domain

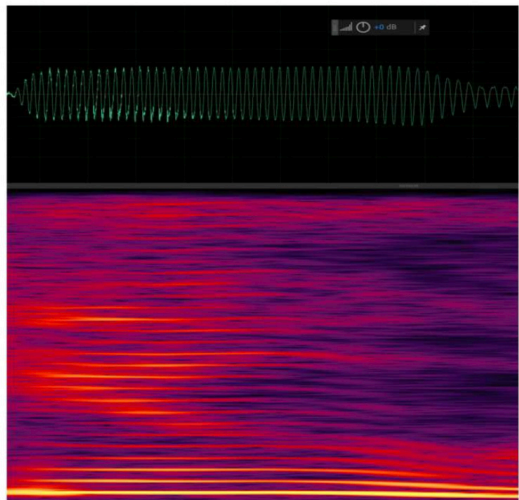
Frequency response

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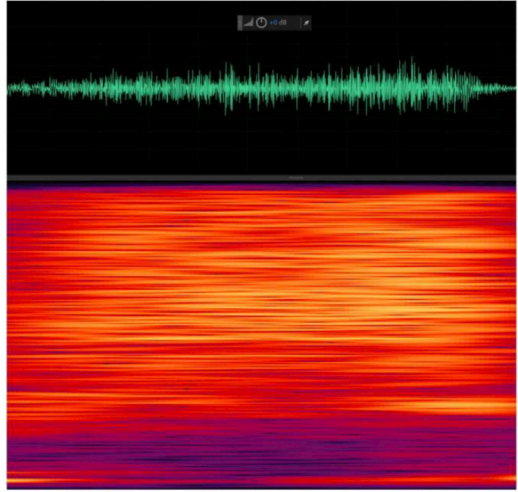


Time and Time-Frequency domain

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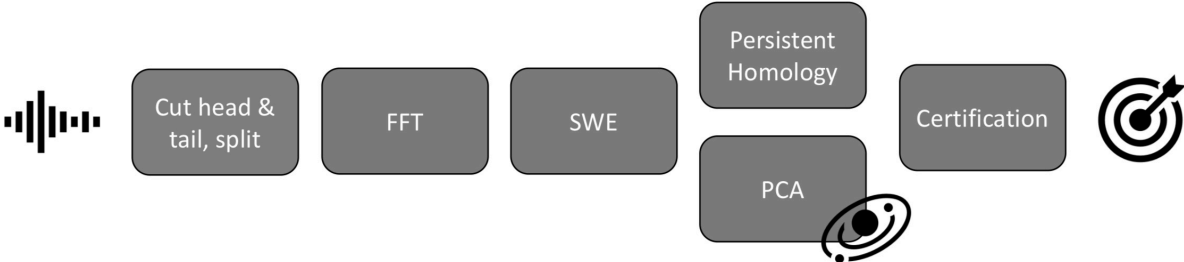
Voiceless

Like a white noise



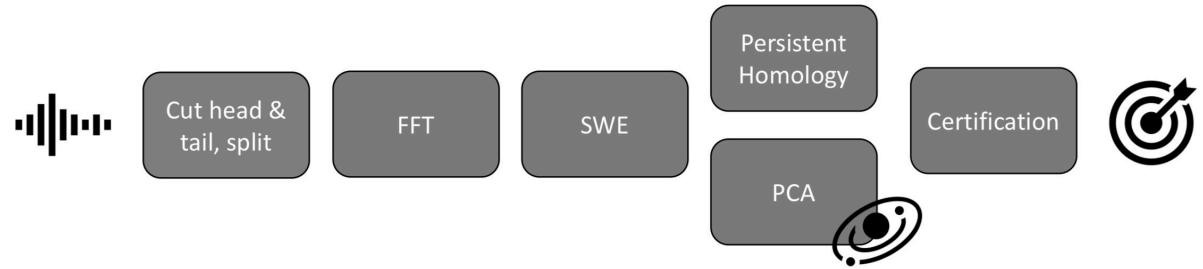
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Here is a flowchart for our topological approach:

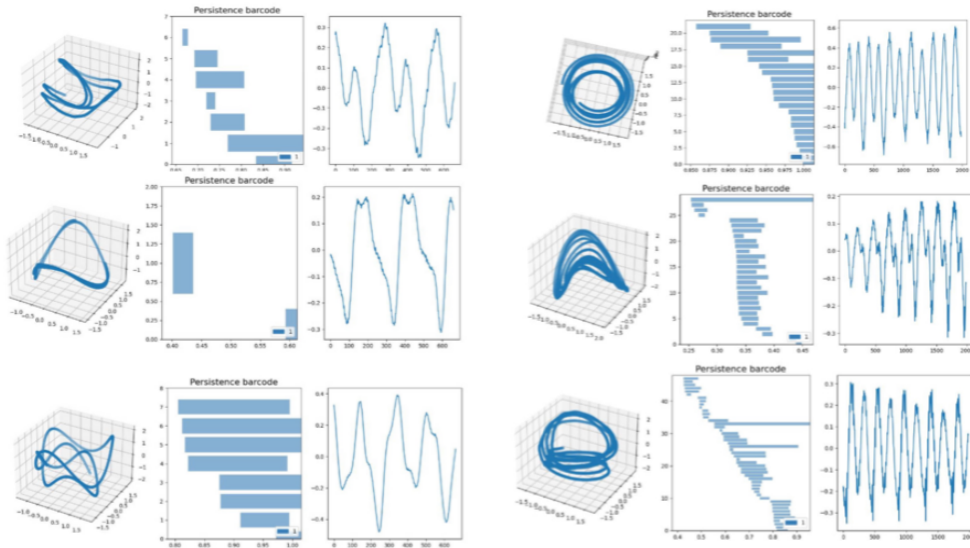


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Topological profiles for vowels and consonants (Pingyao Feng)

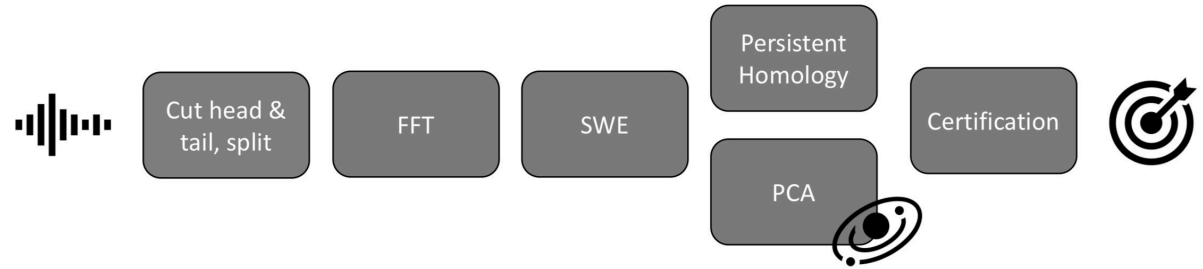


Features for vowels

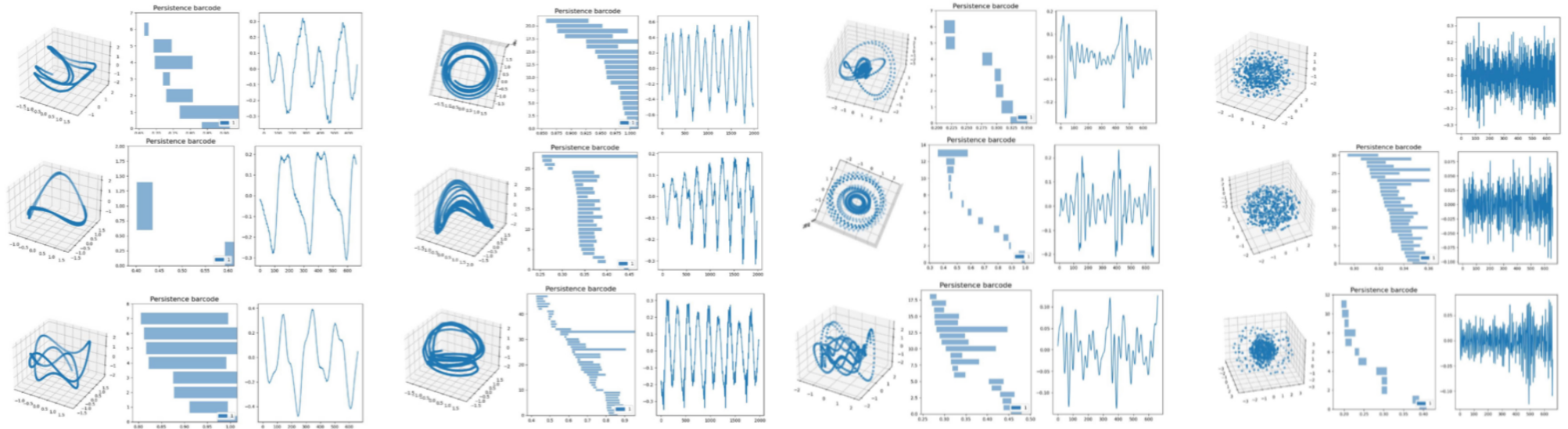
Left: frame size: 15ms, frame shift: 5ms; Right: frame size: 45ms, frame shift: 22.5ms

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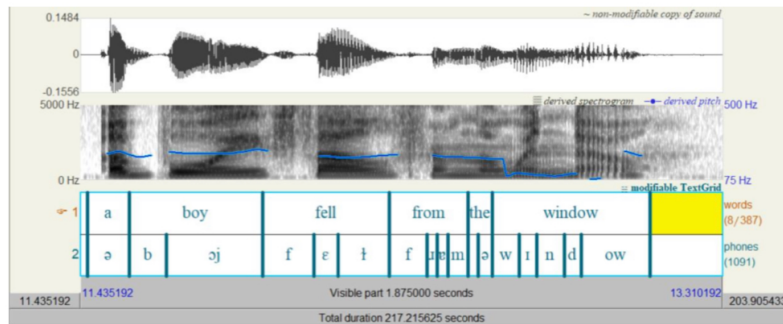
Left: pulmonic consonant; Right: non-pulmonic consonant

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Using real-world speech data from the MFA **aligner**, our research group (Feng) further fed the topological features for **machine learning**, and obtained positive preliminary results for classification.

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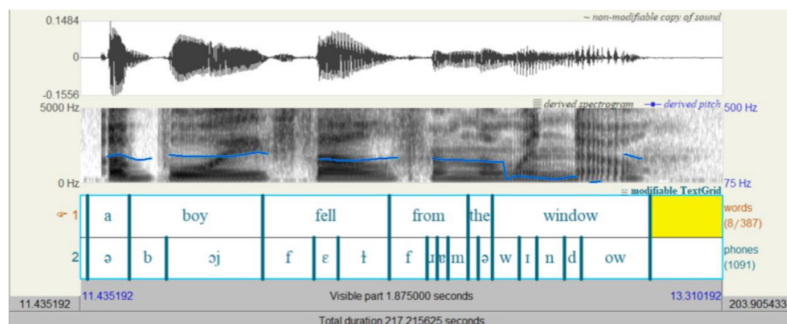
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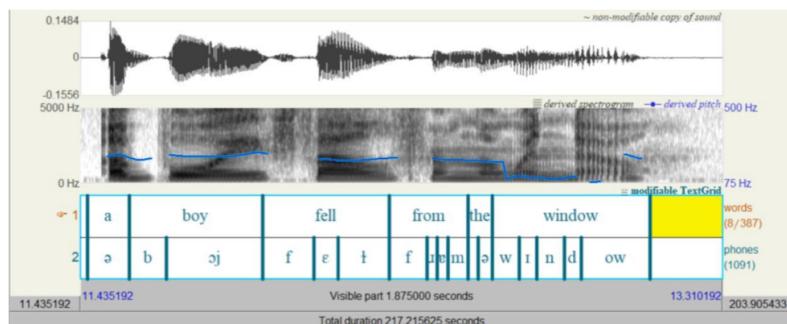
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Last change: Optimizable Tree	10/10 features
6 Ensemble	Accuracy (Validation): 77.1%
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1 Tree	Accuracy (Validation): 75.0%
Last change: Fine Tree	10/10 features
5 KNN	Accuracy (Validation): 75.0%
Last change: Optimizable KNN	10/10 features
8 Tree	Accuracy (Validation): 75.0%
Last change: Medium Tree	10/10 features
3 Optimizable Discr...	Accuracy (Validation): 72.9%
Last change: Optimizable Discriminant	10/10 features
4 SVM	Accuracy (Validation): 70.8%
Last change: Optimizable SVM	10/10 features
7 Neural Network	Accuracy (Validation): 70.8%
Last change: Optimizable Neural Network	10/10 features
9 KNN	Accuracy (Validation): 66.7%
Last change: Hyperparameter option(s)	10/10 features

32 vowels, 16 consonants.
10 features: 5 are barcodes
number of 5 diag, other 5
are number of barcodes that
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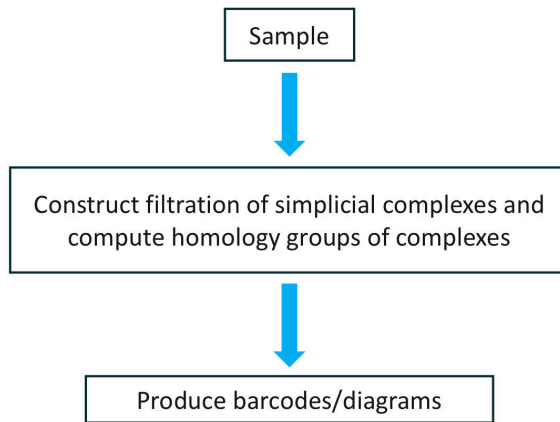
1 Tree	Accuracy (Validation): 81.5%
Last change: Fine Tree	4/4 features
2 Tree	Accuracy (Validation): 81.5%
Last change: Optimizable Tree	4/4 features
7 Tree	Accuracy (Validation): 81.5%
Last change: Medium Tree	4/4 features
4 Tree	Accuracy (Validation): 78.5%
Last change: Coarse Tree	4/4 features
3 KNN	Accuracy (Validation): 69.2%
Last change: Optimizable KNN	4/4 features
5 Neural Network	Accuracy (Validation): 46.2%
Last change: Hyperparameter option(s)	4/4 features
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Last change: Narrow Neural Network	4/4 features

32 vowels, 33 consonants. 4
features: bottleneck distance
between neighborhood
barcode(currently the best
result)

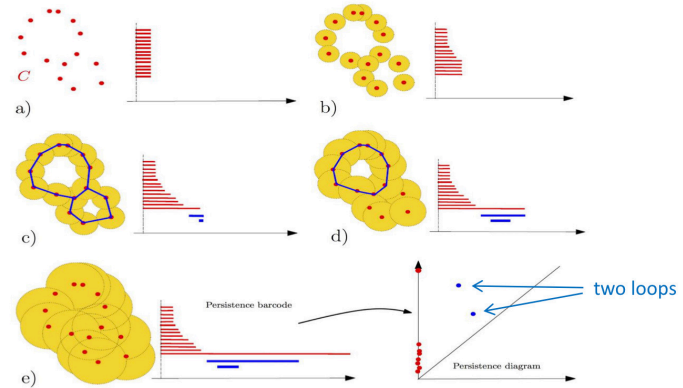
A formal recap of the topological methods applied

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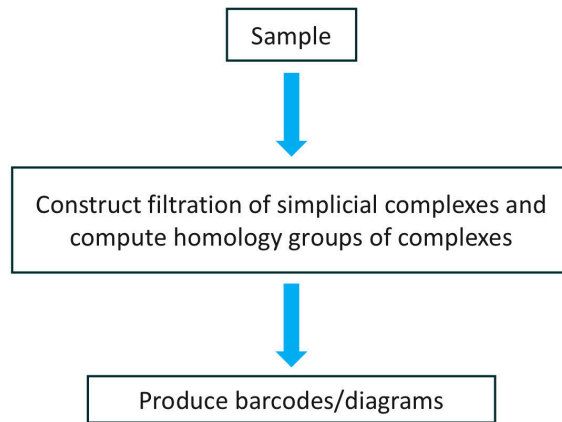


How filtration through varying distance measure reveals essential topological features

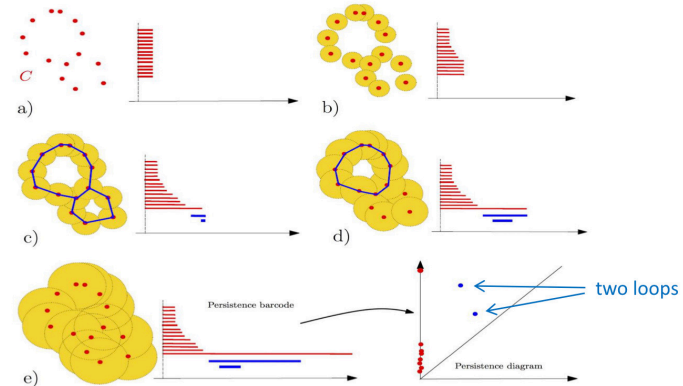


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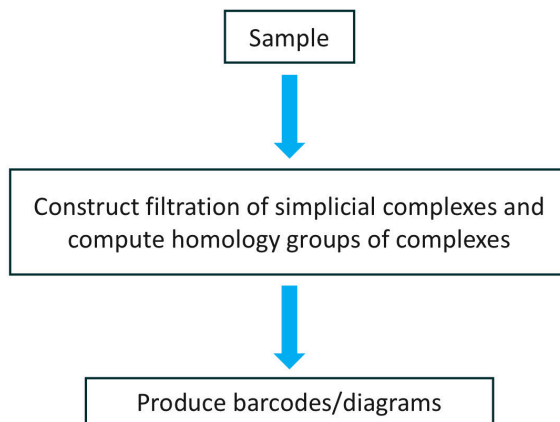


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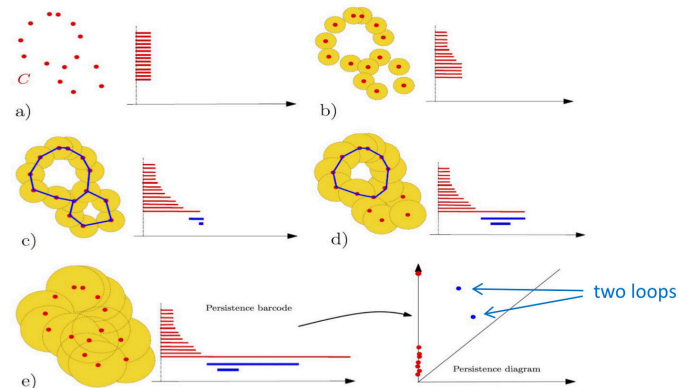
Euclidean embedding of time series data dates back to Takens's work on fluid turbulence in the 1980s.

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Theorem (Takens 1981). Let M be a compact manifold of dimension n . Given pairs (ϕ, y) with $\phi: M \rightarrow M$ a smooth diffeomorphism and $y: M \rightarrow \mathbb{R}$ a smooth function, it is a generic property that the map $\Phi_{(\phi, y)}: M \rightarrow \mathbb{R}^{2n+1}$ defined by

$$\Phi_{(\phi, y)}(x) = \left(y(x), y(\phi(x)), \dots, y(\phi^{2n}(x)) \right)$$

is an **embedding**.

From topological data analysis to topological deep learning

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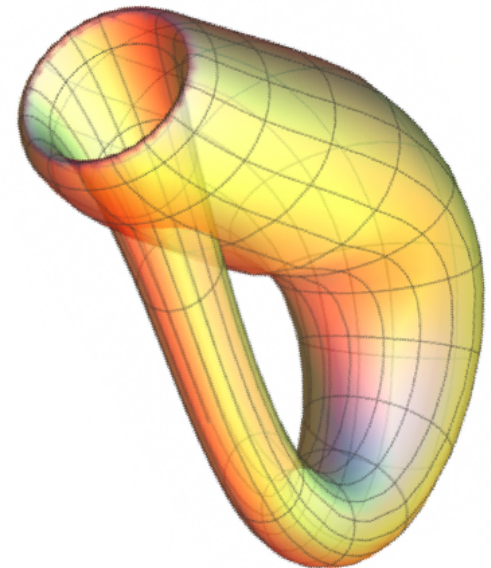
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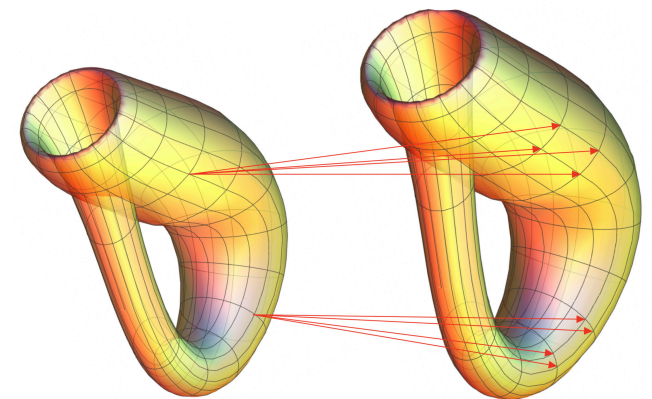
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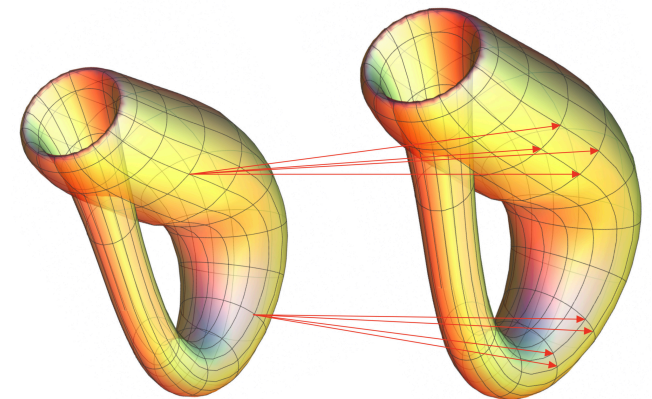
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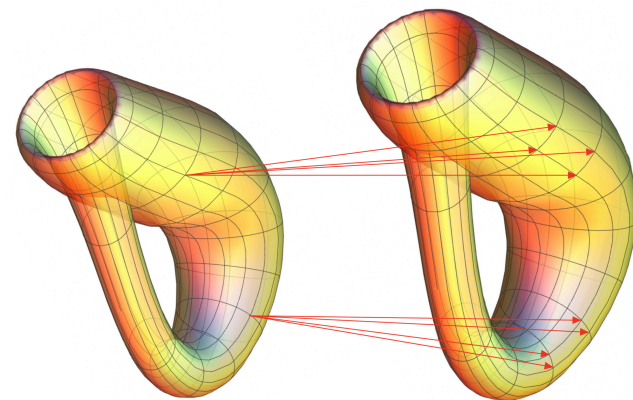
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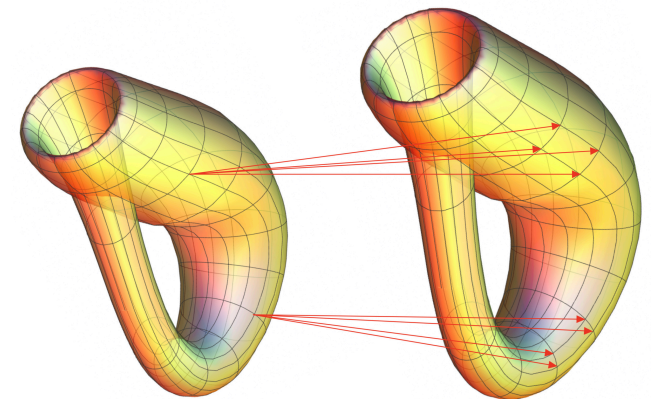


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As a second warm-up, our research group (Zhiwang Yu) have reproduced some of their results.



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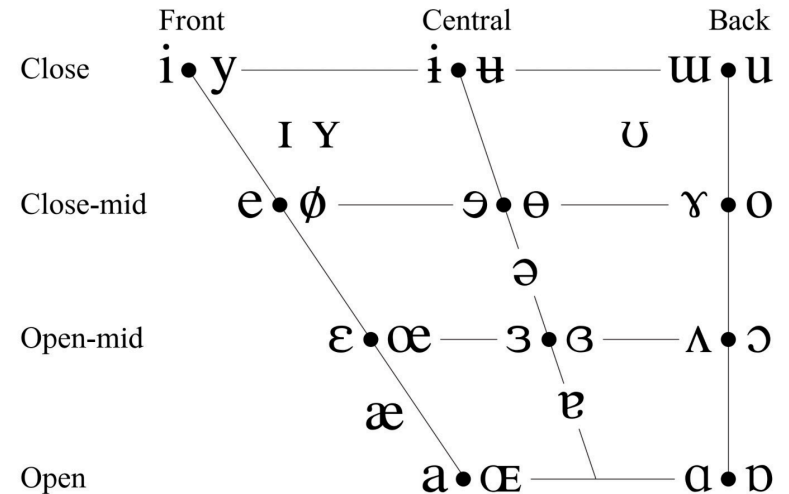
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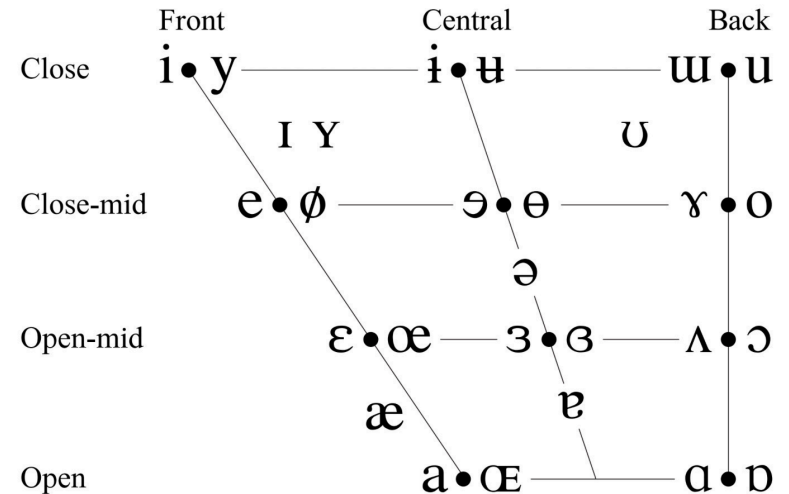


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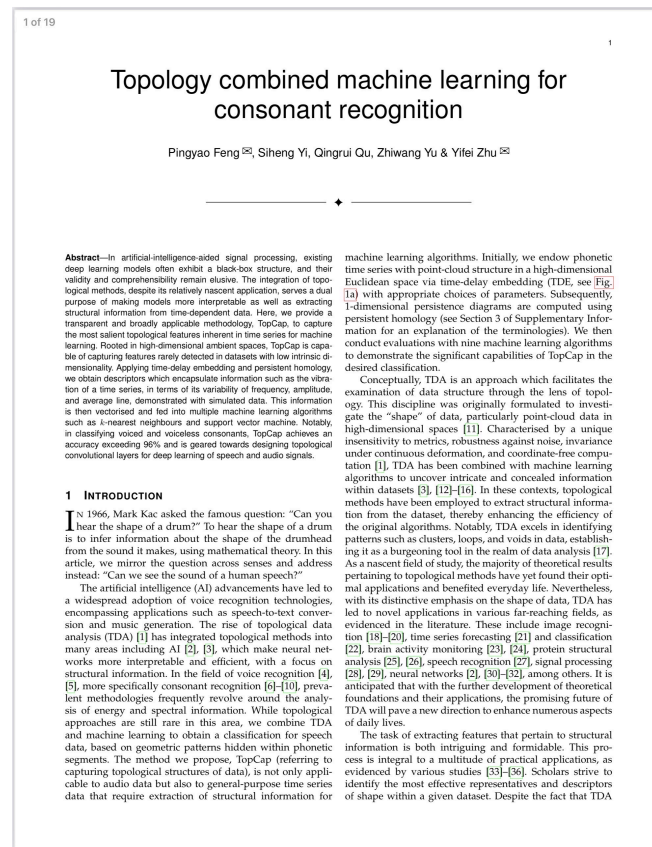
The vertical axis of the chart denotes vowel height. Vowels pronounced with the tongue lowered are at the bottom and raised are at the top. The horizontal axis of the chart denotes vowel backness. Vowels with the tongue moved towards the front of the mouth are in the left of the chart, while those with the tongue moved to the back are placed in right. The last parameter is whether the lips are rounded. At each given spot, vowels on the right and left are rounded and unrounded, respectively.



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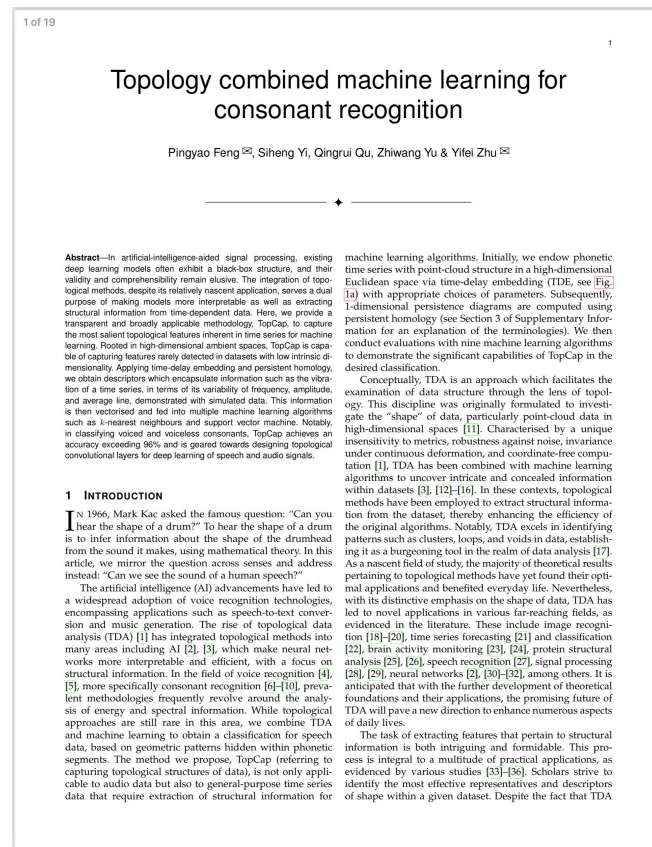
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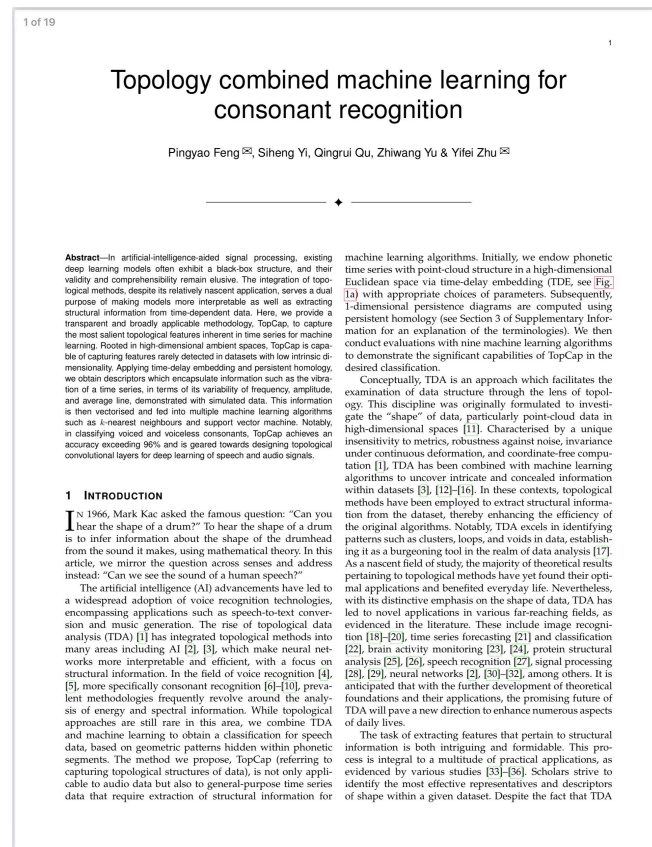
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Topology combined machine learning for consonant recognition

Pingyao Feng[✉], Siheng Yi, Qingrui Ou, Zhiwang Yu & Yifei Zhu[✉]

Abstract—In artificial-intelligence-aided signal processing, existing deep learning models often exhibit a black-box structure, and their validity and comprehensibility remain elusive. The integration of topological methods, despite its relatively nascent application, serves a dual purpose of making models more interpretable as well as extracting structural information from time-dependent data. Here, we provide a transparent and broadly applicable methodology, TopCap, to capture the most salient topological features inherent in time series for machine learning. Rooted in high-dimensional ambient spaces, TopCap is capable of capturing features rarely detected in datasets with low intrinsic dimensionality. Applying time-delay embedding and persistent homology, we obtain descriptors which encapsulate information such as the vibration of a time series, in terms of its variability of frequency, amplitude, and average line, demonstrated with simulated data. This information is then vectorised and fed into multiple machine learning algorithms such as k-nearest neighbours and support vector machine. Notably, in classifying voiced and voiceless consonants, TopCap achieves an accuracy exceeding 96% and is geared towards designing topological convolutional layers for deep learning of speech and audio signals.

machine learning algorithms. Initially, we embed phonetic time series with point-cloud structure in a high-dimensional Euclidean space via time-delay embedding (TDE, see Fig. 1a) with appropriate choices of parameters. Subsequently, 1-dimensional persistence diagrams are computed using persistent homology (see Section 3 of Supplementary Information for an explanation of the terminology). We then conduct evaluations with nine machine learning algorithms to demonstrate the significant capabilities of TopCap in the desired classification.

Conceptually, TDA is an approach which facilitates the examination of data structure through the lens of topology. This discipline was originally formulated to investigate the “shape” of data, particularly point-cloud data in high-dimensional spaces [1]. Characterised by a unique insensitivity to metrics, robustness against noise, invariance under continuous deformation, and coordinate-free computation [1], TDA has been combined with machine learning algorithms to uncover intricate and concealed information within datasets [3], [12]–[16]. In these contexts, topological methods have been employed to extract structural information from the dataset, thereby enhancing the efficiency of the original algorithms. Notably, TDA excels in identifying patterns such as clusters, loops, and voids in data, establishing it as a burgeoning tool in the realm of data analysis [17]. As a nascent field of study, the majority of theoretical results pertaining to topological methods have yet found their optimal applications and benefited everyday life. Nevertheless, with its distinctive emphasis on the shape of data, TDA has led to novel applications in various far-reaching fields, as evidenced in the literature. These include image recognition [18]–[20], time series forecasting [21] and classification [22], brain activity monitoring [23], [24], protein structural analysis [25], [26], speech recognition [27], signal processing [28], [29], neural networks [2], [30]–[32], among others. It is anticipated that with the further development of theoretical foundations and their applications, the promising future of TDA will pave a new direction to enhance numerous aspects of daily lives.

The task of extracting features that pertain to structural information is both intriguing and formidable. This process is integral to a multitude of practical applications, as evidenced by various studies [33]–[36]. Scholars strive to identify the most effective representatives and descriptors of shape within a given dataset. Despite the fact that TDA

1 INTRODUCTION

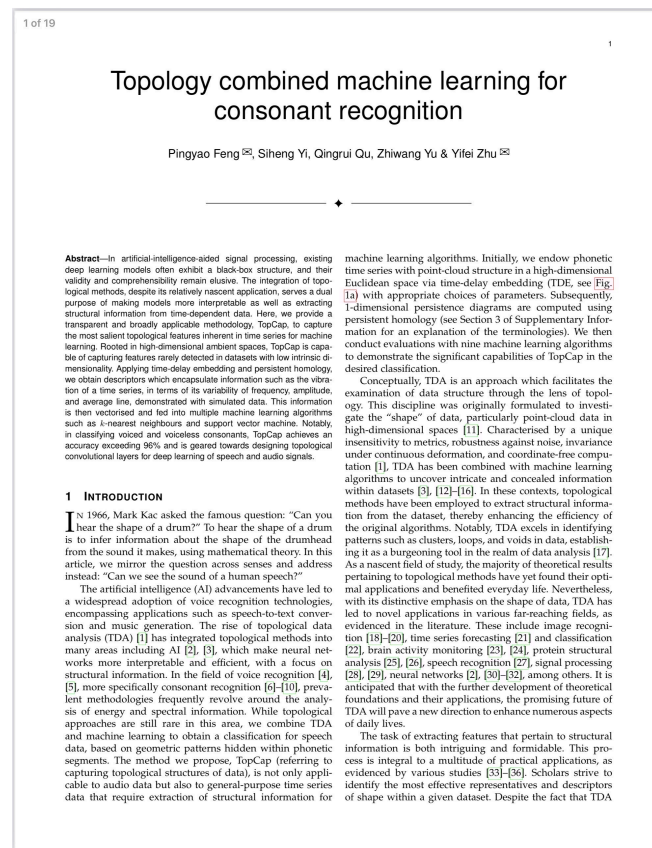
IN 1966, Mark Kac asked the famous question: “Can you hear the shape of a drum?” To hear the shape of a drum is to infer information about the shape of the drumhead from the sound it makes, using mathematical theory. In this article, we mirror the question across senses and address instead: “Can we see the sound of a human speech?”

The artificial intelligence (AI) advancements have led to a widespread adoption of voice recognition technologies, encompassing applications such as speech-to-text conversion and music generation. The rise of topological data analysis (TDA) [1] has integrated topological methods into many areas including AI [2], [3], which make neural networks more interpretable and efficient, with a focus on structural information. In the field of voice recognition [4], [5], more specifically consonant recognition [6]–[10], prevalent methodologies frequently revolve around the analysis of energy and spectral information. While topological approaches are still rare in this area, we combine TDA and machine learning to obtain a classification for speech data, based on geometric patterns hidden within phonetic segments. The method we propose, TopCap (referring to capturing topological structures of data), is not only applicable to audio data but also to general-purpose time series data that require extraction of structural information for

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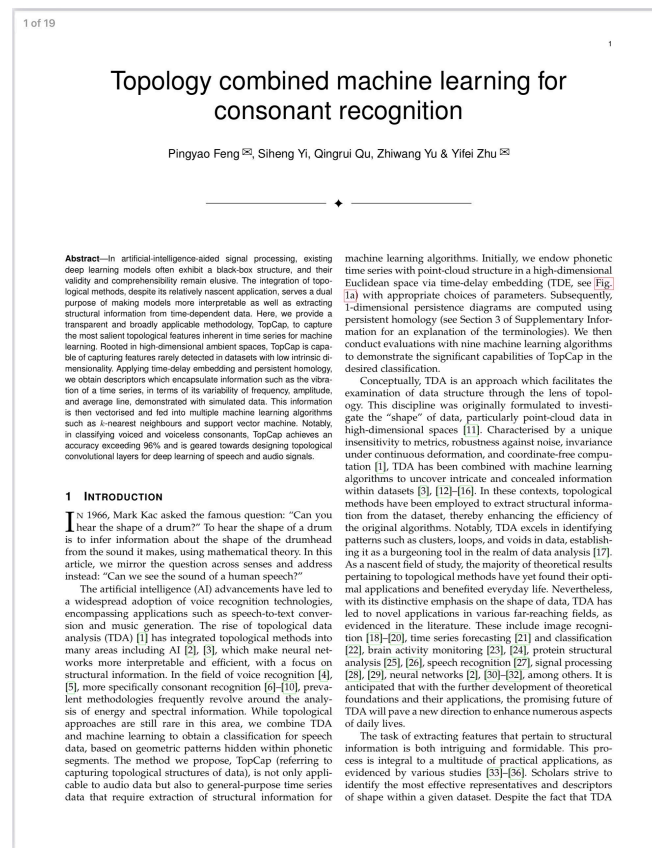
For phonetic data, linguists created a charted “distribution space” of vowels. Using speech files from SpeechBox, our topological approach achieved **an average accuracy exceeding 96% in classifying voiced and voiceless consonants** via machine learning. A main goal remains to use topological methods to reveal a **distribution space for speech data**, even a **digraph** on it modeling the **complex network of speech-signal sequences**, and apply these topological inputs for **smarter learning**.



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nature
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ARTICLES

PUBLISHED ONLINE: 12 JUNE 2017 | DOI: 10.1038/NPHOTON.2017.93

Deep learning with coherent nanophotonic circuits

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Artificial neural networks are computational network models inspired by signal processing in the brain. These models have dramatically improved performance for many machine-learning tasks, including speech and image recognition. However, today's computing hardware is inefficient at implementing neural networks, in large part because much of it was designed for von Neumann computing schemes. Significant effort has been made towards developing electronic architectures tuned to implement artificial neural networks that exhibit improved computational speed and accuracy. Here, we propose a new architecture for a fully optical neural network that, in principle, could offer an enhancement in computational speed and power efficiency over state-of-the-art electronics for conventional inference tasks. We experimentally demonstrate the essential part of the concept using a programmable nanophotonic processor featuring a cascaded array of 56 programmable Mach-Zehnder interferometers in a silicon photonic integrated circuit and show its utility for vowel recognition.

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It will be useful to design and fine-tune them topologically (joint with Huan Li of optical science and engineering at Zhejiang University and Xinxiang Niu of Huawei).

Thank you.