Study Note on Article:

Topology of Musical Data

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Part I

Symmetric Product and Bundles

1. The n-fold *symmetric product* of X is the quotient of X^n, by the action of the symmetric group $\sum_n$ (unorder, as well sub-symmetric product $Symm_n^G(X)$).

2. Bundle is a triple of the form (E, p, B), where p: $E \to B$. E is called totle space and B is called base, p is the projection of the bundle (E, p, B). The inverse image $p^{-1}(b)$ of a point b ∈ B is called the fibre bundle over the point b.

Example: Some regular surface; Standard trivial bundle $(B \times F, pr_1, B)$ (definition of trivial bundle); Disk bundle: fibre is a disk.

3. Morton's Bundle: $Symm_n(S^1)$ is a disk bundle over S^1 which is orientable iff n is odd. Projection p given by multiple, convex set.

Part II

Vietoris Rips complex

Vietoris Rips complex: A complex that built from point cloud and metric. To be more concrete: Given a distance δ, there is a rips complex correspond formed by the distance δ, suth that k point formed a k-1 complex iff each pair in this k point have dimension $< \delta$.

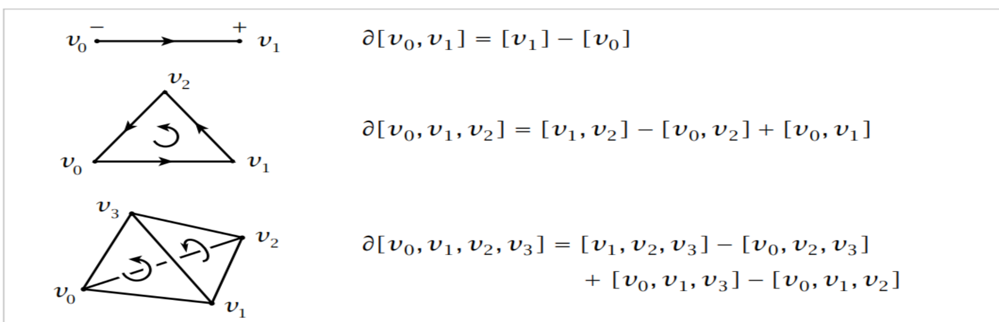
Remark: The construction of Rips complex is closely related to metric on the space. Compare with CW complex, simplicial complex and abstract simplicial complex. (Also Δ-complex)

Part III

Presistent Homology

Homology: how cells of dimension n attach to cells of dimension n-1.

1. Simplicial homology: boundary homeomorphism; Why a proper definition?; $H_n^{\Delta}(X) = Ker\partial_n / Im\partial_{n+1}$.



In the last case, the orientations of the two hidden faces are also counterclockwise when viewed from outside the 3-simplex.

With this geometry in mind we define for a general Δ-complex X a **boundary homomorphism** $\partial_n : \Delta_n(X) \to \Delta_{n-1}(X)$ by specifying its values on basis elements:

$$\partial_n(\sigma_\alpha) = \sum_i (-1)^i \sigma_\alpha | [v_0, \cdots, \hat{v}_i, \cdots, v_n]$$

Figure 1: from A.H p.105

2. Singular homology: Singular n-simplex

Part IV

Distance Measure in musical data

1. Euclidean measure in $\mathbb{S}^1$ (or so called nacklace distance)

2. To improve: Adding logarithmic to the measure (used in time delay to improve the dimension of data)

3. To improve on increasing dimensional: “modulo permutation”

Part V

Conclusions and Discussion